

Statistical Mechanics of Money, Income, and Wealth

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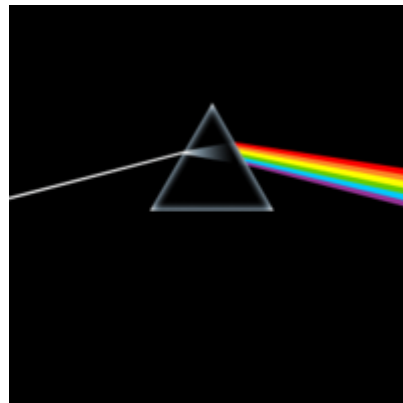
- *European Physical Journal B* **17**, 723 (2000) → →
- *Reviews of Modern Physics* **81**, 1703 (2009)
- Book *Classical Econophysics* (Routledge, 2009)
- Submitted to *New Journal of Physics*, arXiv:0912.4898 (2009)

Outline of the talk

- Statistical mechanics of money
- Debt and financial instability
- Two-class structure of income distribution
- Global inequality in energy consumption

“Money, it’s a gas.”

Pink Floyd



Boltzmann-Gibbs versus Pareto distribution



Ludwig Boltzmann (1844-1906)

Boltzmann-Gibbs probability distribution
 $P(\varepsilon) \propto \exp(-\varepsilon/T)$, where ε is energy, and
 $T = \langle \varepsilon \rangle$ is temperature.



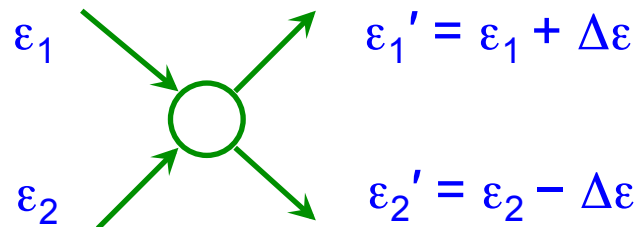
Vilfredo Pareto (1848-1923)

Pareto probability distribution
 $P(r) \propto 1/r^{(\alpha+1)}$ of income r .

An **analogy** between the distributions of **energy** ε and **money** m or **income** r

Boltzmann-Gibbs probability distribution of **money**

Collisions between atoms



Conservation of energy:

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$$

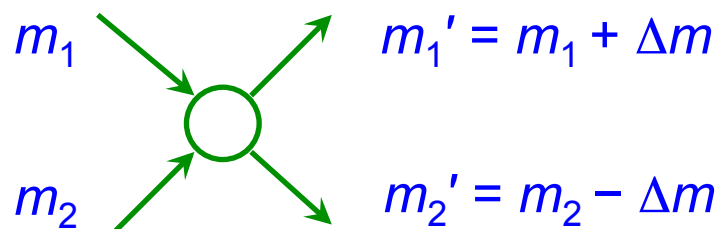
Detailed balance:

~~$$w_{12 \rightarrow 1'2'} P(\varepsilon_1) P(\varepsilon_2) = w_{1'2' \rightarrow 12} P(\varepsilon_1') P(\varepsilon_2')$$~~

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε , where $T = \langle \varepsilon \rangle$ is temperature. It is **universal** – independent of model rules, provided the model belongs to the **time-reversal symmetry** class.

Boltzmann-Gibbs distribution **maximizes entropy** $S = -\sum_{\varepsilon} P(\varepsilon) \ln P(\varepsilon)$ under the constraint of conservation law $\sum_{\varepsilon} P(\varepsilon) \varepsilon = \text{const.}$

Economic transactions between agents



Conservation of money:

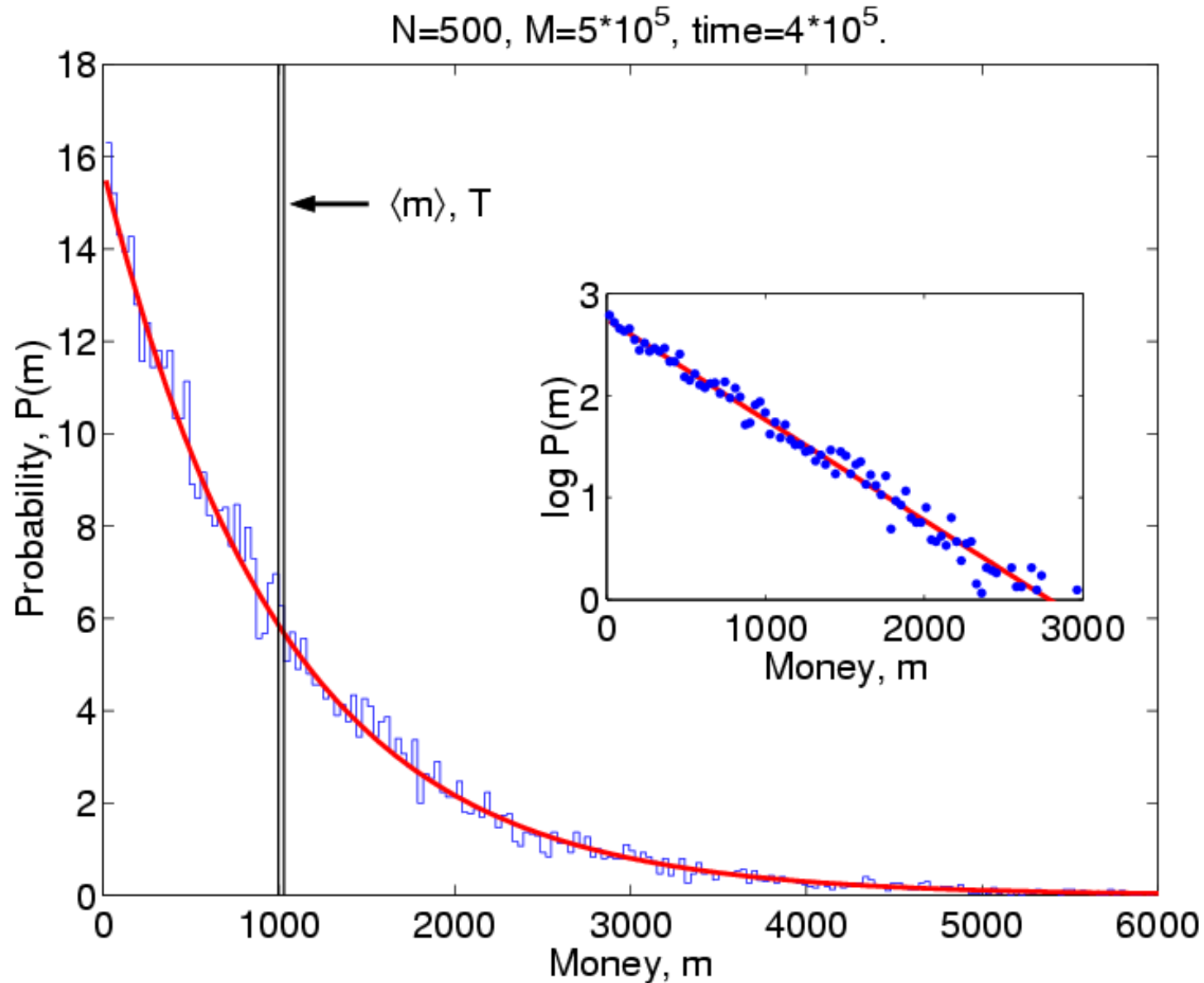
$$m_1 + m_2 = m_1' + m_2'$$

Detailed balance:

$$w_{12 \rightarrow 1'2'} P(m_1) P(m_2) = w_{1'2' \rightarrow 12} P(m_1') P(m_2')$$

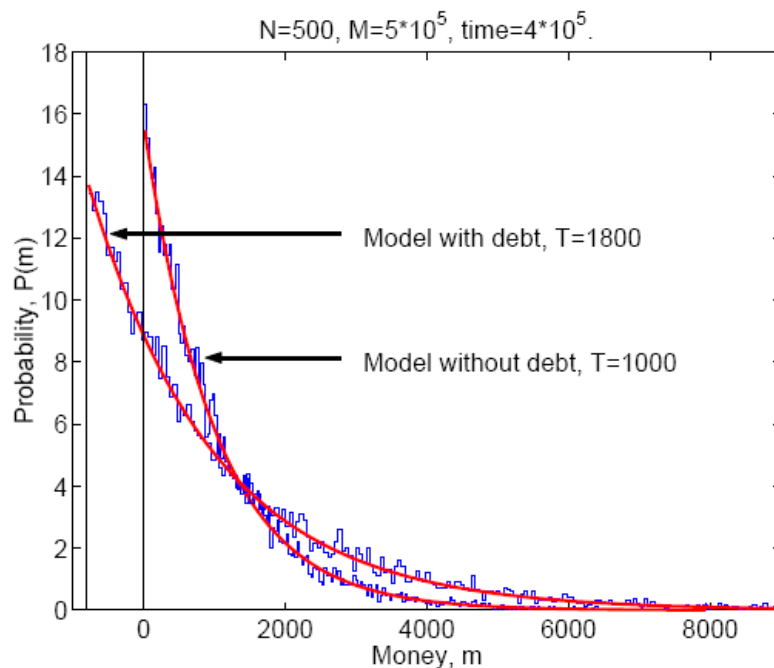
Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of **money** m , where $T = \langle m \rangle$ is the **money temperature**.

Computer simulation of money redistribution

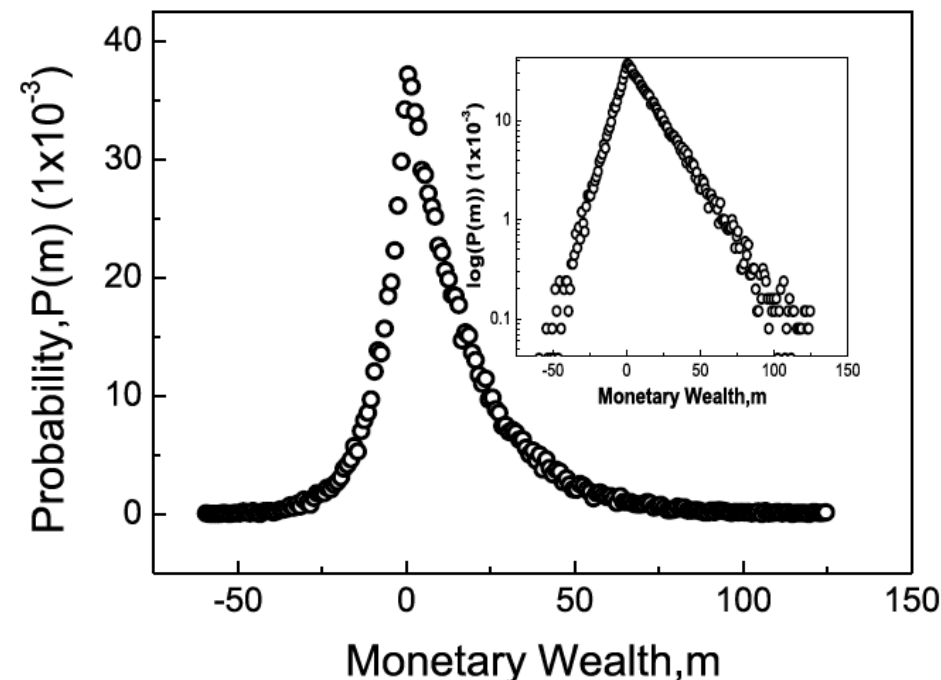


The stationary distribution of money m is exponential:
 $P(m) \propto e^{-m/T}$

Money distribution with debt



Debt per person is limited to 800 units.

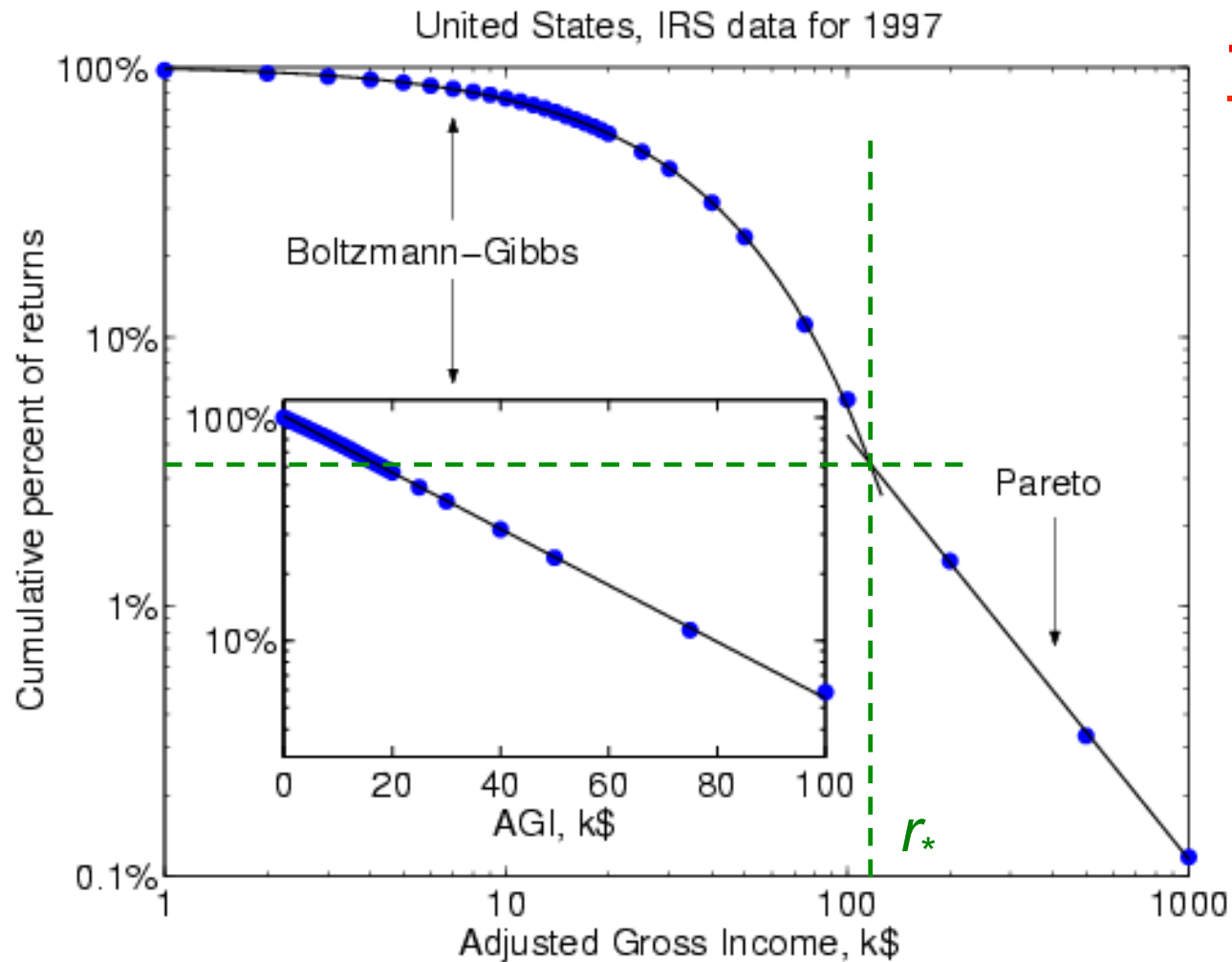


Total debt in the system is limited via the Required Reserve Ratio (RRR):

Xi, Ding, Wang, *Physica A* **357**, 543 (2005)

- In practice, RRR is enforced inconsistently and does not limit total debt.
- Without a constraint on debt, the system does **not have a stationary equilibrium**.
- Free market itself does not have an intrinsic mechanism for limiting debt, and there is **no such thing as the equilibrium debt**.

Income distribution in the USA, 1997



Two-class society

Upper Class

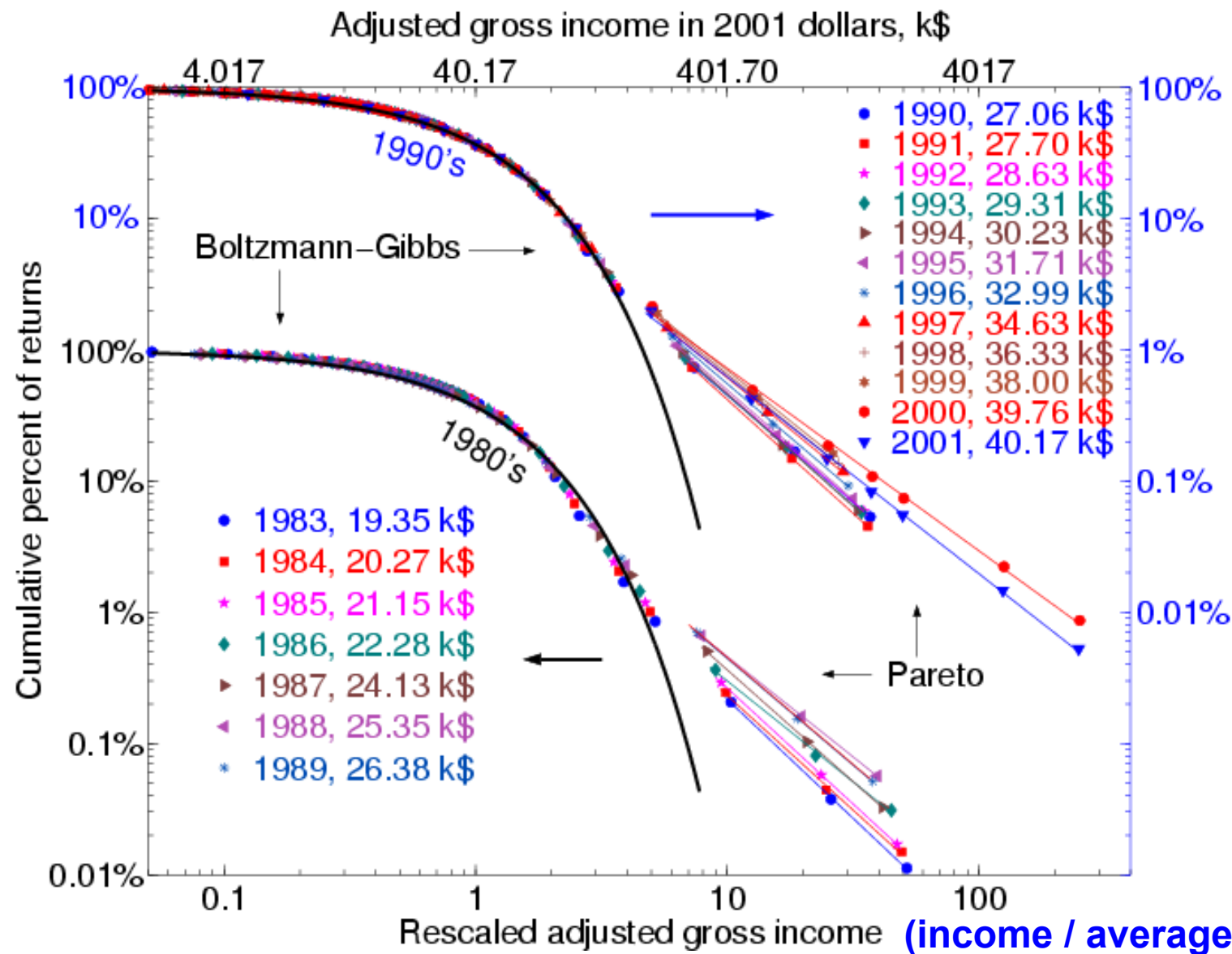
- Pareto power law
- 3% of population
- 16% of income
- Income > 120 k\$: investments, capital

Lower Class

- Boltzmann-Gibbs exponential law
- 97% of population
- 84% of income
- Income < 120 k\$: wages, salaries

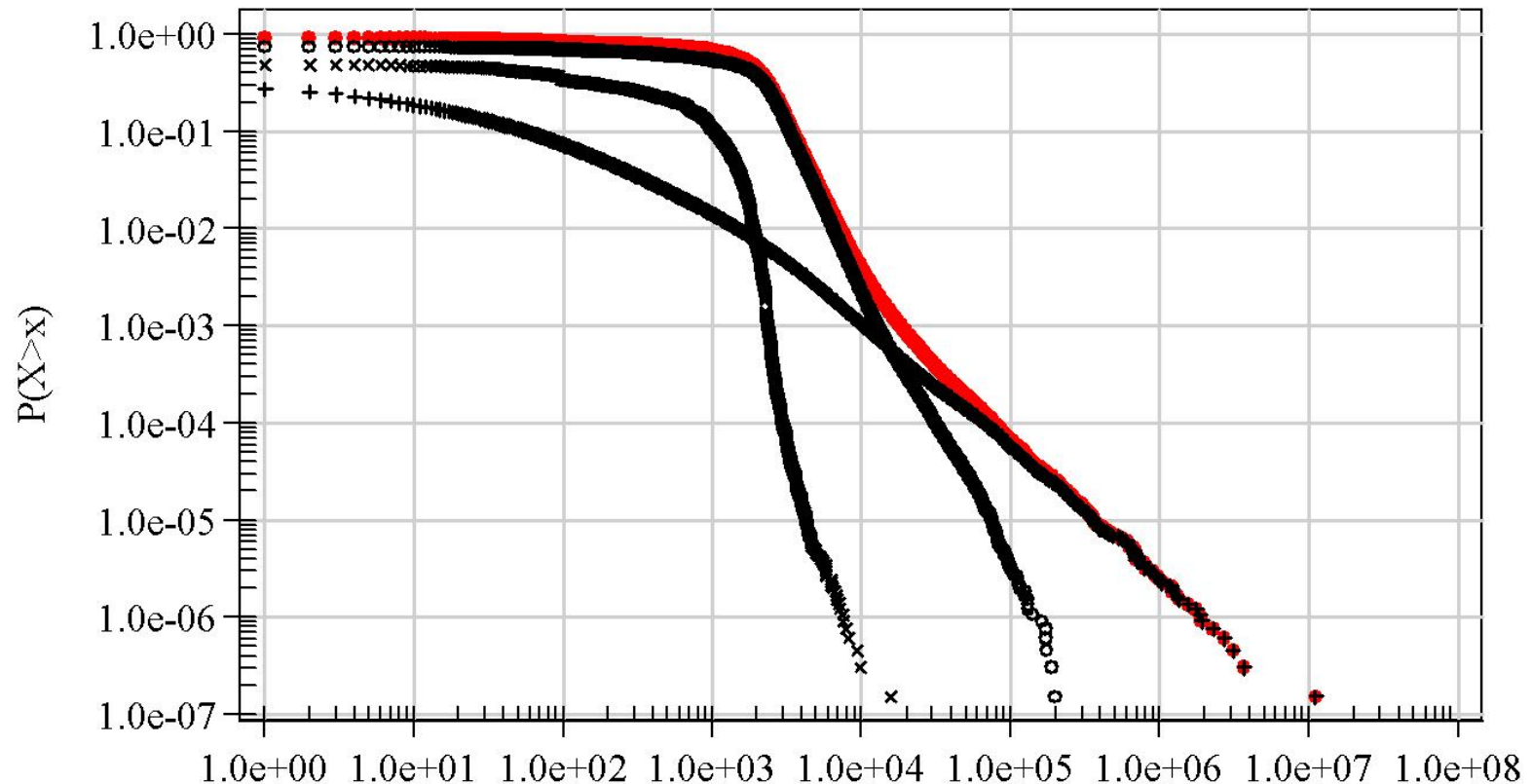
“Thermal” bulk and “super-thermal” tail distribution

Income distribution in the USA, 1983-2001



The rescaled
exponential
part does not
change,
but the
power-law
part changes
significantly.

Income distribution in Sweden



The data plot from
Fredrik Liljeros and Martin Hällsten,
Stockholm University

- Total incomes
- Work
- + Capital
- × Social transfers

The origin of two classes

- Different sources of income: salaries and wages for the lower class, and capital gains and investments for the upper class.
- Their income dynamics can be described by additive and multiplicative diffusion, correspondingly.
- From the social point of view, these can be the classes of employees and employers, as described by Karl Marx.
- Emergence of classes from the initially equal agents was simulated by Ian Wright “The Social Architecture of Capitalism” *Physica A* **346**, 589 (2005), see also the new book “Classical Econophysics” (2009)

Diffusion model for income kinetics

Suppose income changes by small amounts Δr over time Δt . Then $P(r,t)$ satisfies the Fokker-Planck equation for $0 < r < \infty$:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r} \left(AP + \frac{\partial}{\partial r} (BP) \right), \quad A = - \left\langle \frac{\Delta r}{\Delta t} \right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t} \right\rangle.$$

For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r} (BP) = -AP$.

For the lower class, Δr are independent of r – additive diffusion, so A and B are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – an exponential distribution.

For the upper class, $\Delta r \propto r$ – multiplicative diffusion, so $A = ar$ and $B = br^2$. Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – a power-law distribution.

For the upper class, income does change in percentages, as shown by Fujiwara, Souma, Aoyama, Kaizoji, and Aoki (2003) for the tax data in Japan. For the lower class, the data is not known yet.

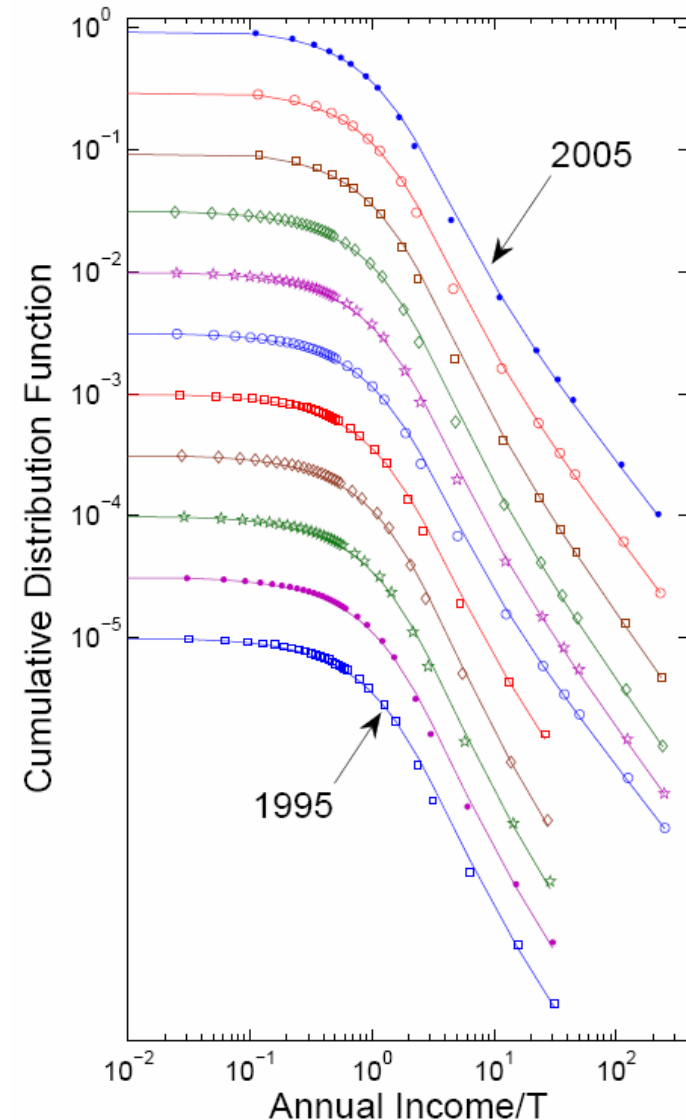
Additive and multiplicative income diffusion

If the **additive** and **multiplicative** diffusion processes are present **simultaneously**, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is

$$P(r) = \frac{C e^{-\frac{r_0}{T} \arctan\left(\frac{r}{r_0}\right)}}{\left[1 + (r/r_0)^2\right]^{1+a/2b}}$$

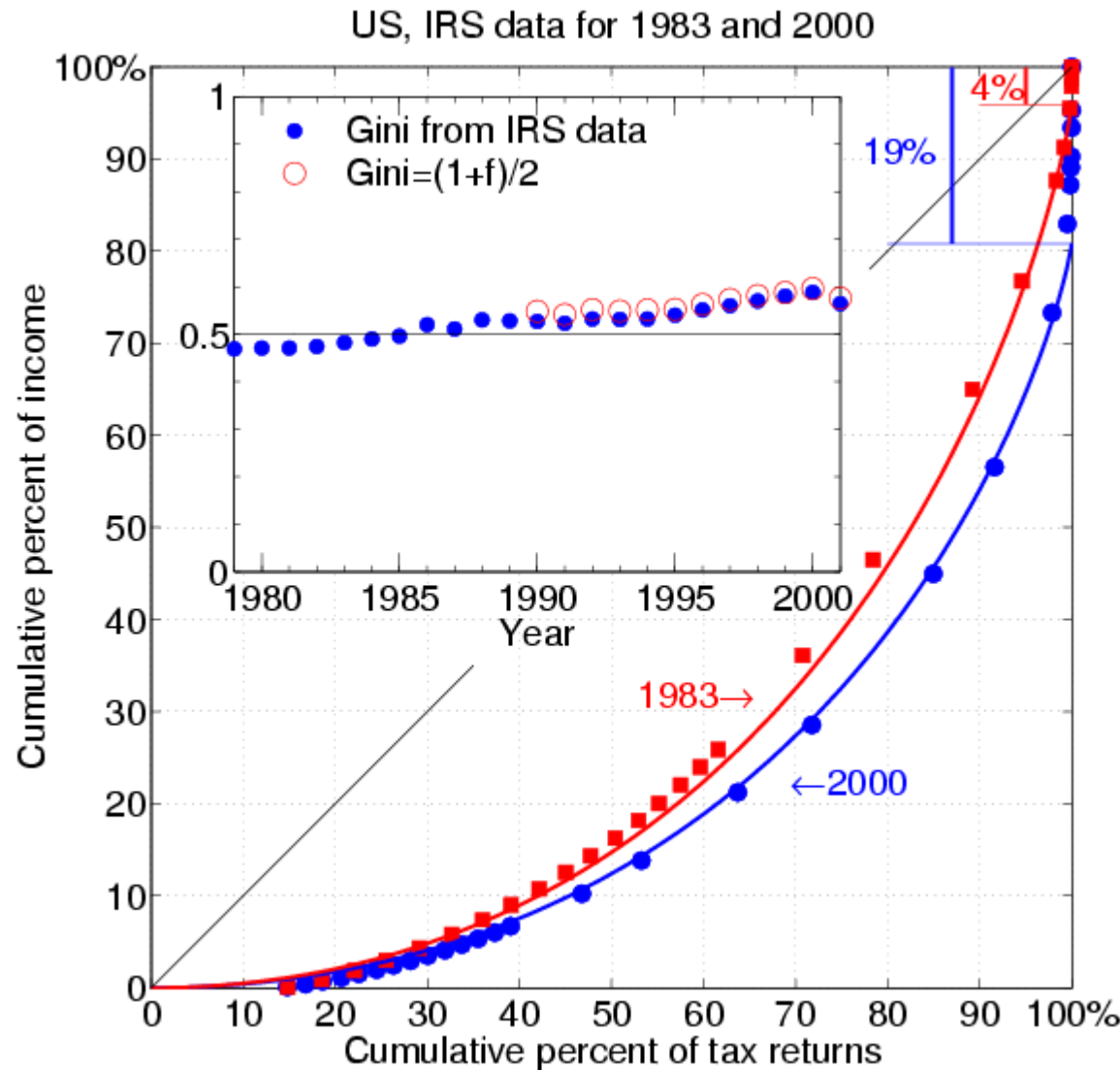
It interpolates between the exponential and the power-law distributions and has 3 parameters:

- $T = B_0/A_0$ – the **temperature** of the exponential part
- $\alpha = 1 + a/b$ – the **power-law exponent** of the upper tail
- r_0 – the **crossover income** between the lower and upper parts.



Yakovenko (2007) arXiv:0709.3662, Fiaschi and Marsili (2007) preprint online

Lorenz curves and income inequality



Lorenz curve ($0 < r < \infty$):

$$x(r) = \int_0^r P(r') dr'$$

$$y(r) = \int_0^r r' P(r') dr' / \langle r' \rangle$$

A measure of inequality,
the **Gini coefficient** is $G =$

$$\frac{\text{Area}(\text{diagonal line} - \text{Lorenz curve})}{\text{Area}(\text{Triangle beneath diagonal})}$$

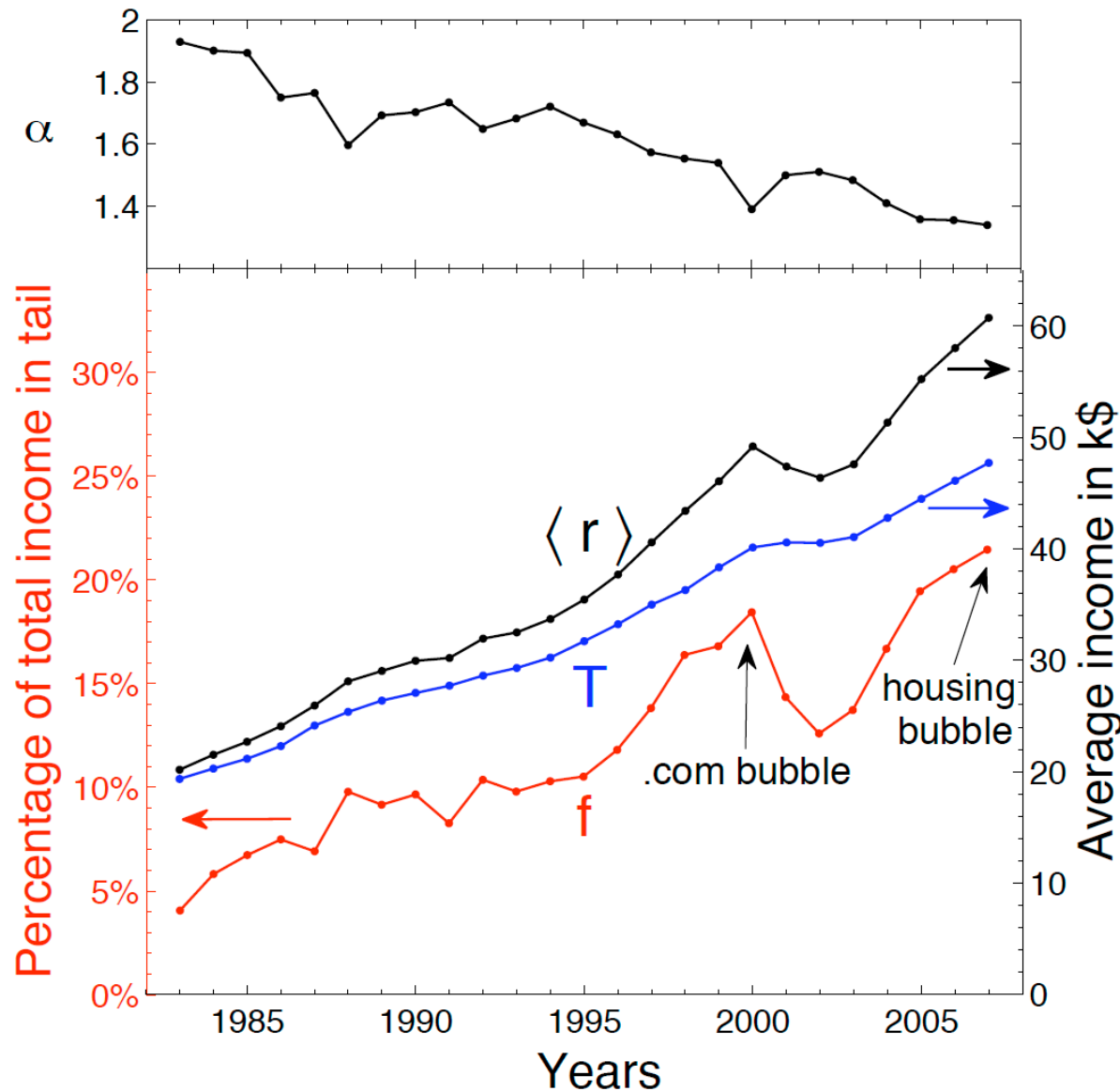
For **exponential distribution**,
 $G = 1/2$ and the **Lorenz curve** is

$$y = x + (1 - x) \ln(1 - x)$$

With a **tail**, the **Lorenz curve** is

$$y = (1 - f)[x + (1 - x) \ln(1 - x)] + f \Theta(x - 1),$$
 where f is the tail income, and
Gini coefficient is $G = (1 + f)/2$.

f - fraction of total income in the tail



$$f = \frac{\langle r \rangle - T}{\langle r \rangle}$$

T – average income in the exponential part

$\langle r \rangle$ – average income in the whole system

Income inequality peaks during speculative bubbles in financial markets

“The next great depression will be from 2008 to 2023”

Harry S. Dent, book “The Great Boom Ahead”, page 16,
published in 1993

His forecast was based on demographic data: The post-war “baby boomers” generation to invest retirement savings in the stock market massively in the 1990s.

His new book “The Great Depression Ahead”, January 2009

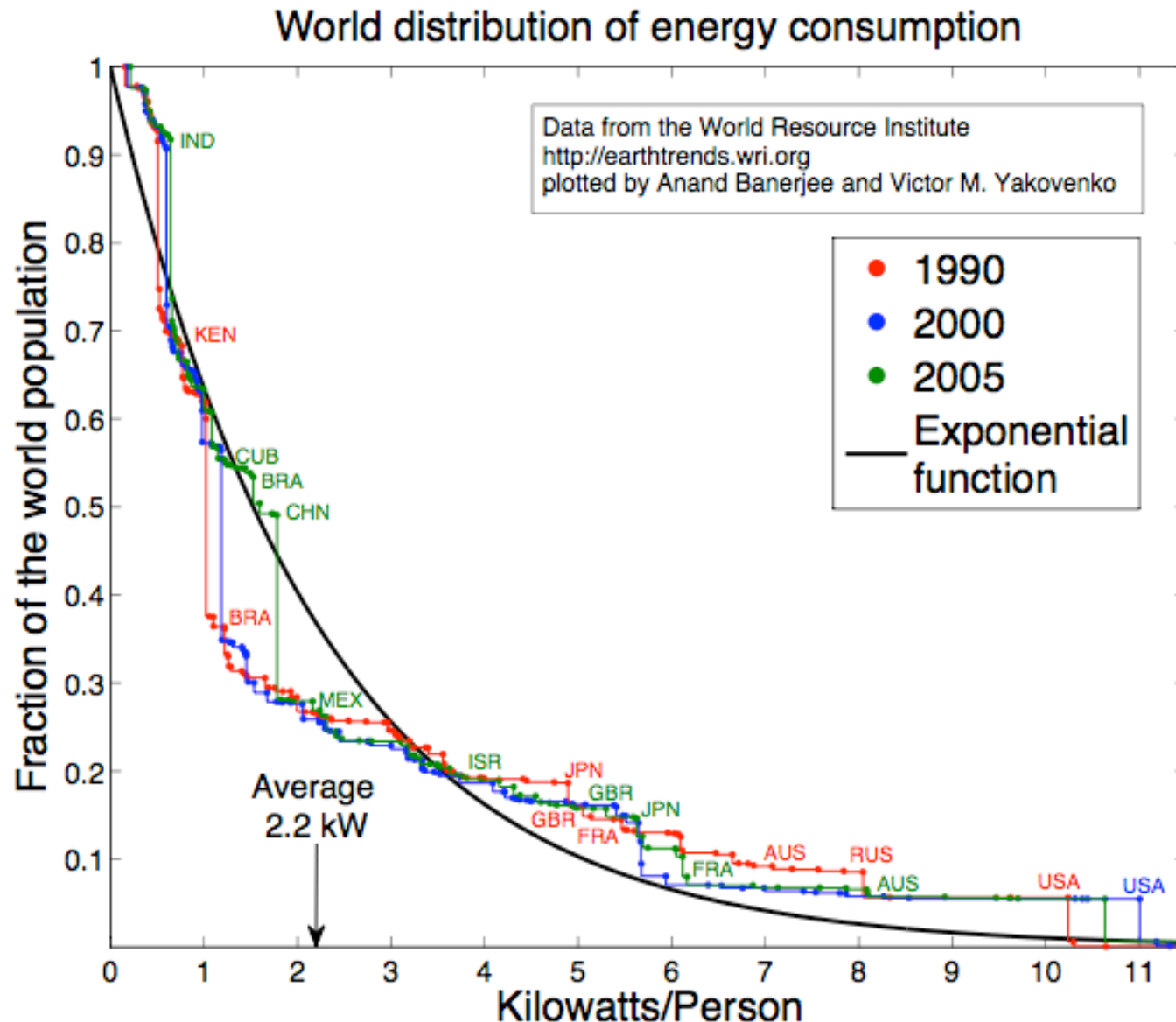
The current financial crisis is not the only and, perhaps, not the most important crises that the mankind faces:

- **exhaustion of fossil fuels** and other natural resources
- **global warming** caused by CO₂ emissions from fossil fuels

Brief history of the **biosphere evolution**:

- **Plants** consume and store energy from the **Sun** through photosynthesis
- **Animals** eat **plants**, which store **Sun's energy**
- **Animals** eat **animals**, which eat **plants**, which store **Sun's energy**
- **Humans** eat **all of the above**,
+ consume **dead plants and animals (fossil fuels)**, which store **Sun's energy**
- For **thousands of years**, the progress of human civilization was **biologically limited** by **muscle energy** (of humans or animals) and by **wood fuel**.
- **Couple of centuries** ago, the humans discovered how to massively utilize Sun's energy stored in **fossil fuels** (coal and oil): the era of **industrial revolution** and **modern capitalism**.
- In a **couple of centuries**, the humans managed to **spend** fossil fuels accumulated for **millions of years**.
- Now this **energy binge** is coming to an **end**. Will humankind manage to find a new way for **sustainable life**? Will **new technology** save us?

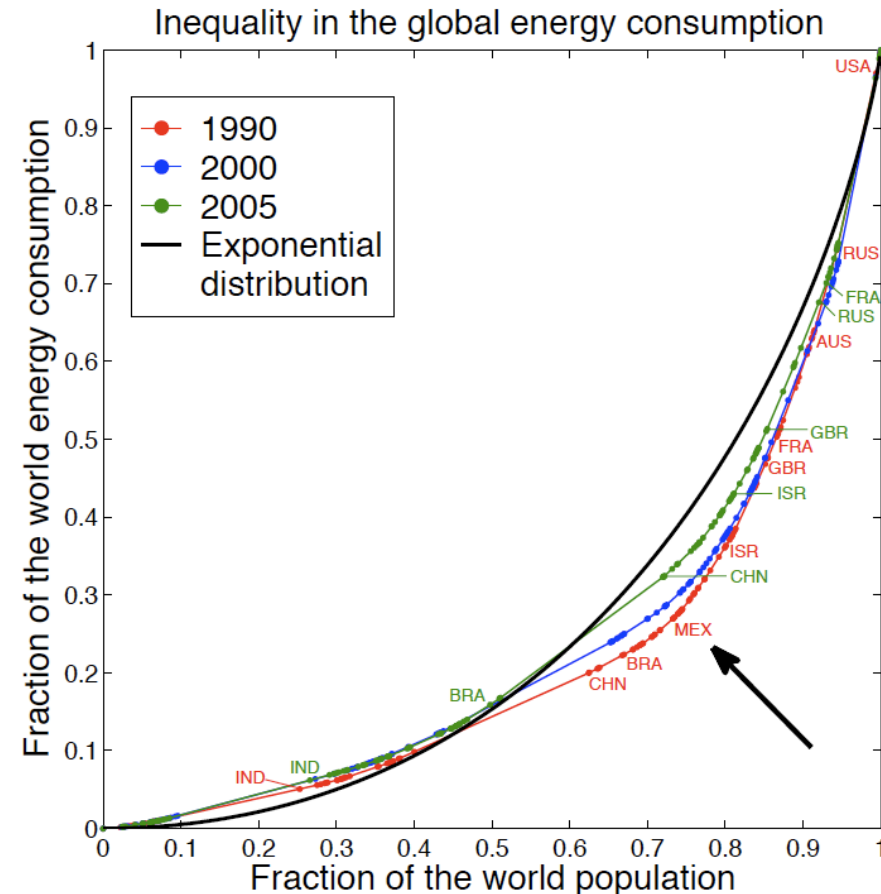
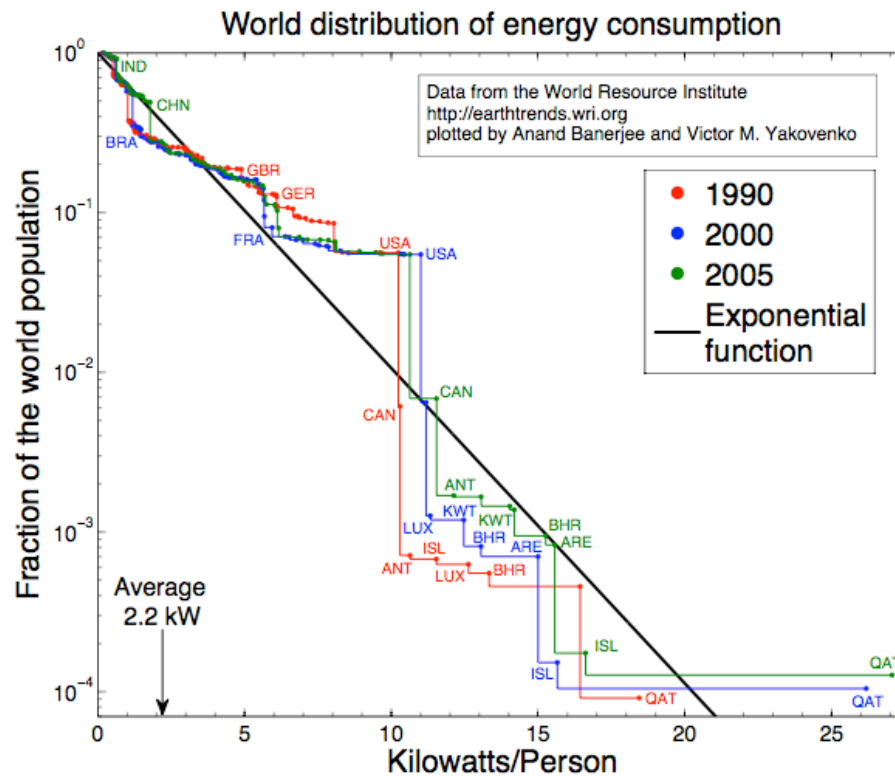
Global inequality in energy consumption



Global distribution of energy consumption per person is roughly exponential.

Physiological energy consumption of a human at rest is about 100 W

Global inequality in energy consumption



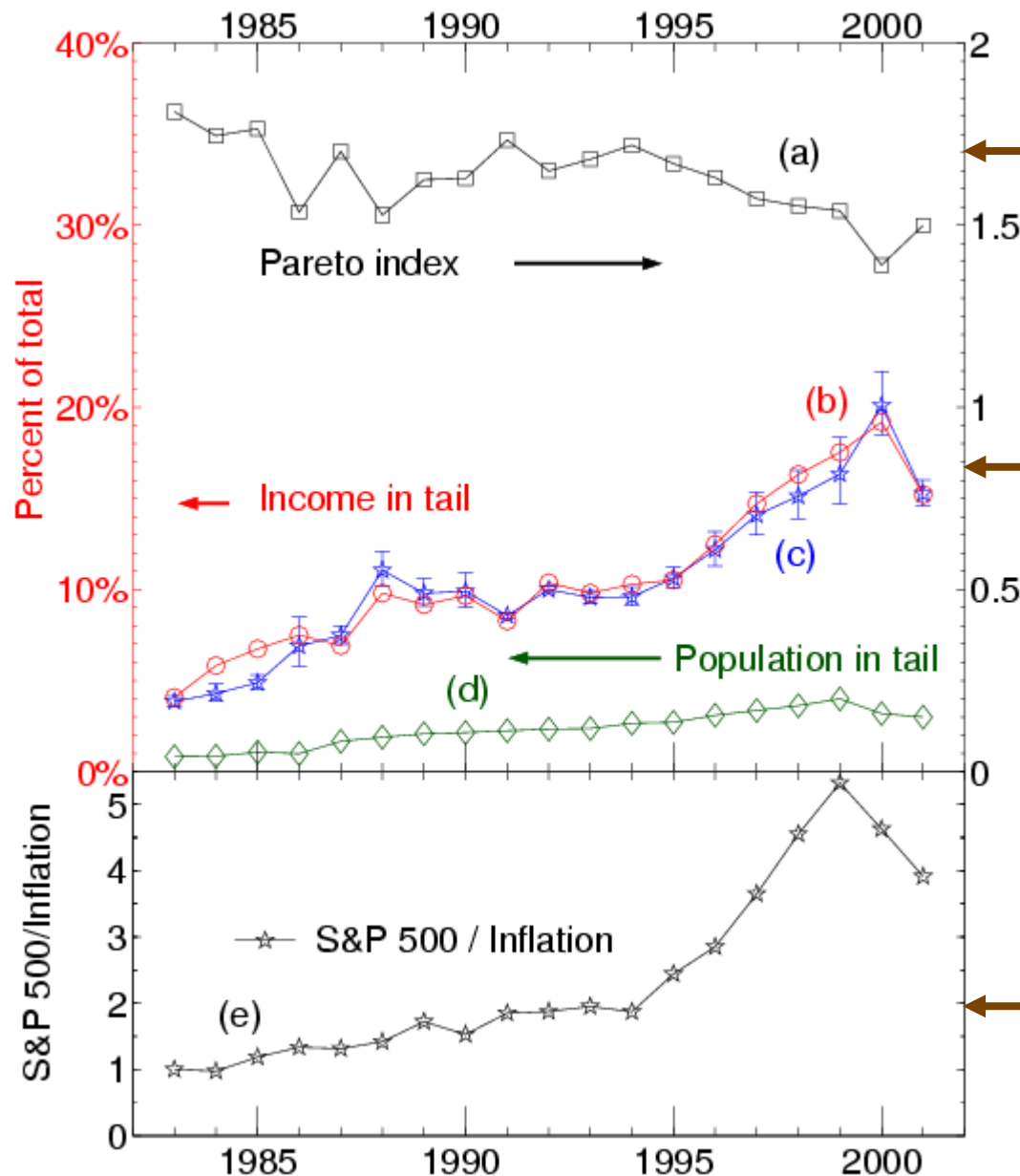
The global distribution of **energy consumption** per person is **highly unequal**. Its **exponential** shape is similar to other **patterns of inequality** (money, income, wealth).

It is also common in **ecology** for partitioning of a **limited resource**.

Conclusions

- The probability **distribution of money** is **stable** and has an **equilibrium** only when a **boundary condition**, such as $m > 0$, is imposed.
- When **debt** is permitted, the distribution of money becomes **unstable**, unless some sort of a **limit on maximal debt** is imposed.
- **Income distribution** in the USA has a **two-class structure**: **exponential** (“thermal”) for the great **majority (97-99%) of population** and **power-law** (“superthermal”) for the **top 1-3% of population**.
- The **exponential part** of the distribution is **very stable** and does not change in time, except for a **slow increase of temperature T** (the average income).
- The **power-law tail** is **not universal** and was increasing significantly for the last 20 years. It peaked and crashed in **2000** and **2006** with the **speculative bubbles** in financial markets.
- The global distribution of **energy consumption** per person is **highly unequal** and **roughly exponential**. This inequality is important in dealing with the global energy problems.

Time evolution of the tail parameters



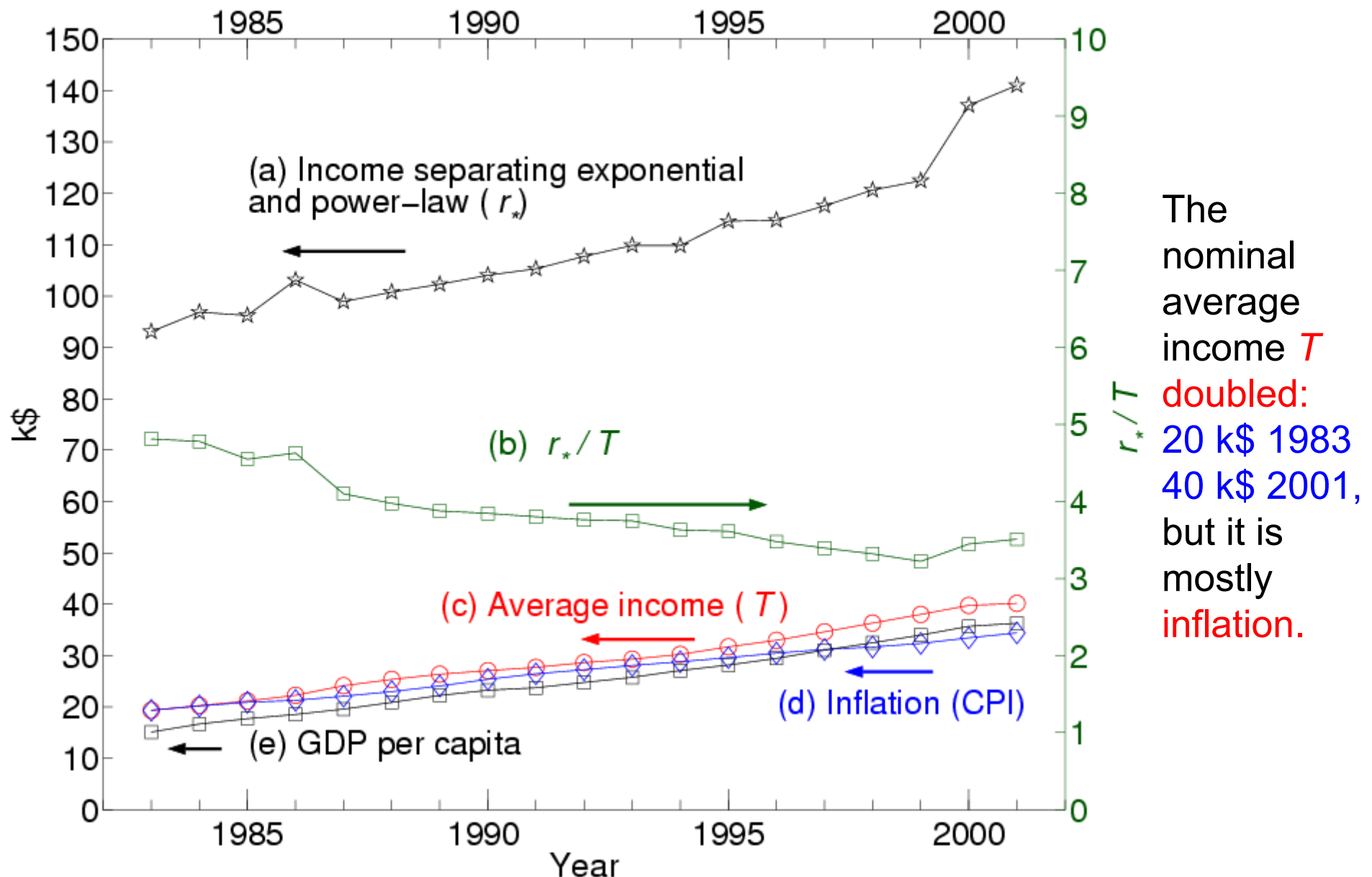
The Pareto index α in $C(r) \propto 1/r^\alpha$ is non-universal. It changed from 1.7 in 1983 to 1.3 in 2000.

- Pareto tail changes in time non-monotonously, in line with the stock market.

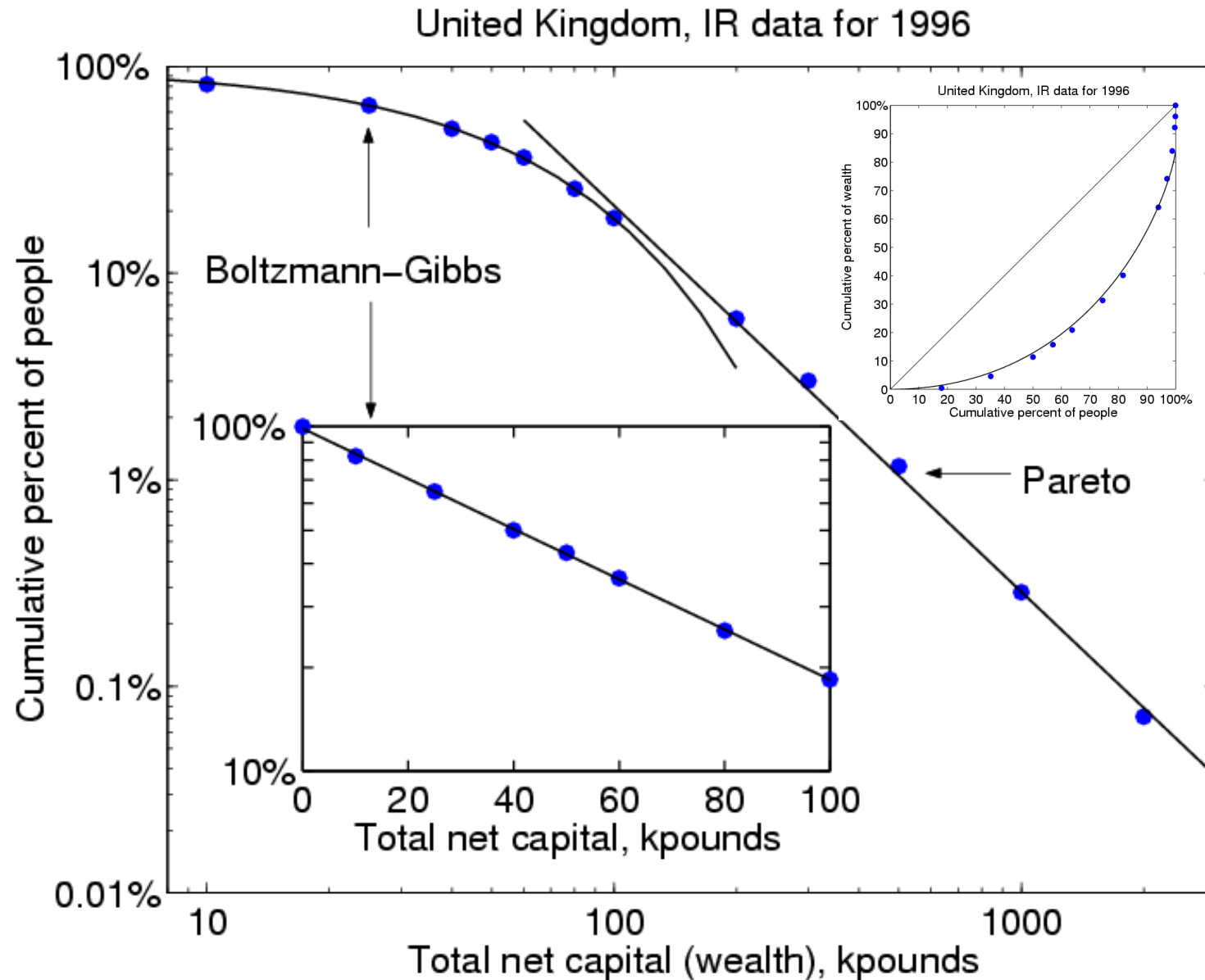
- The tail income swelled 5-fold from 4% in 1983 to 20% in 2000.

- It decreased in 2001 with the crash of the U.S. stock market.

Time evolution of income temperature



Wealth distribution in the United Kingdom

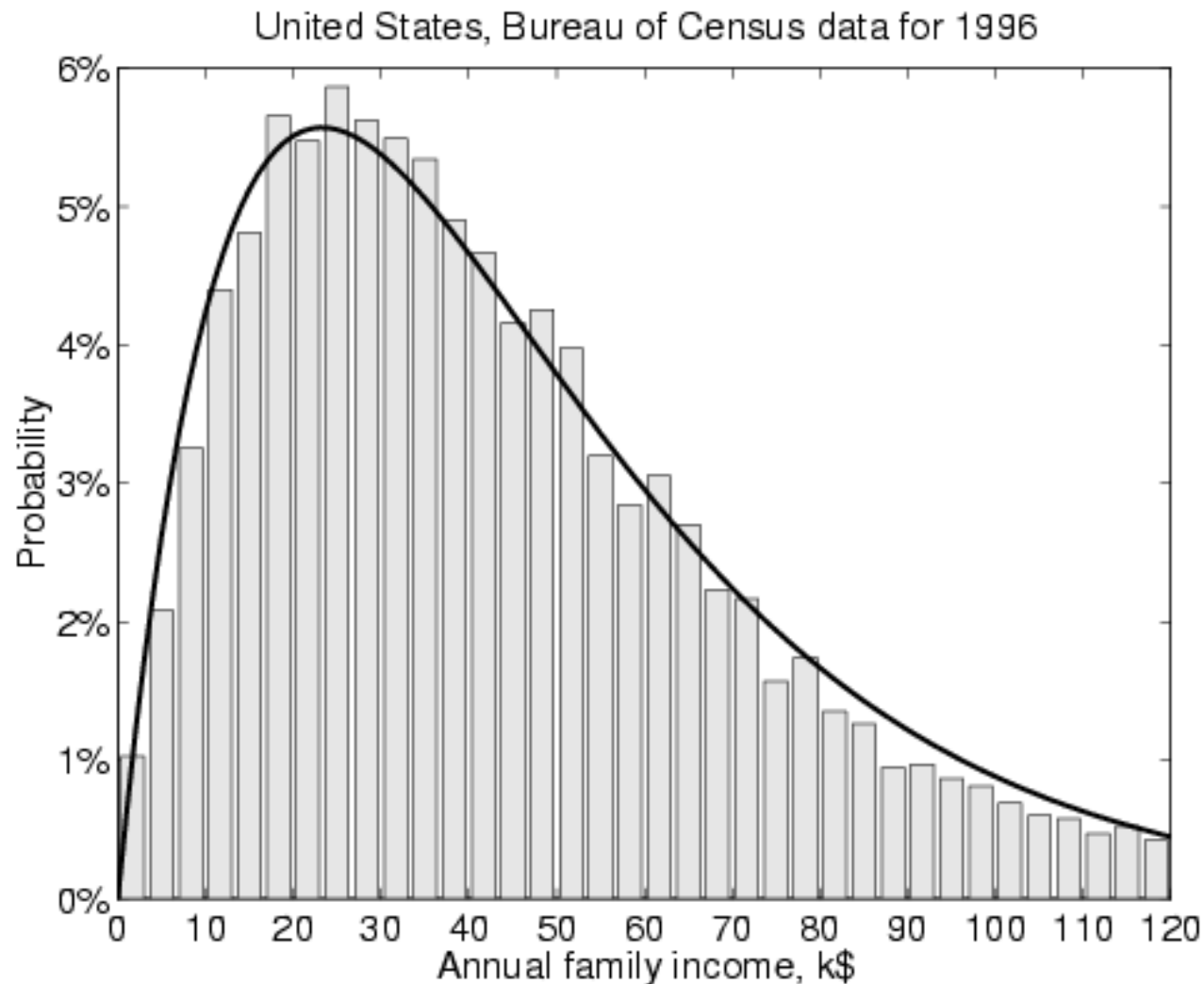


For UK
in 1996,
 $T = 60$ k£

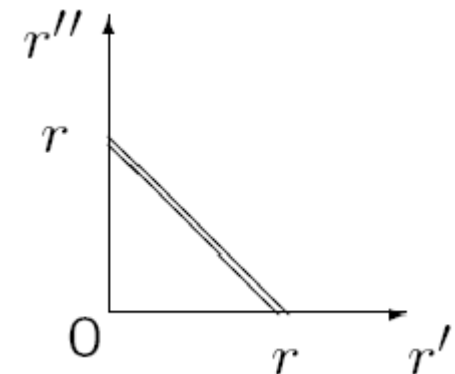
Pareto
index
 $\alpha = 1.9$

Fraction
of wealth
in the tail
 $f = 16\%$

Income distribution for two-earner families

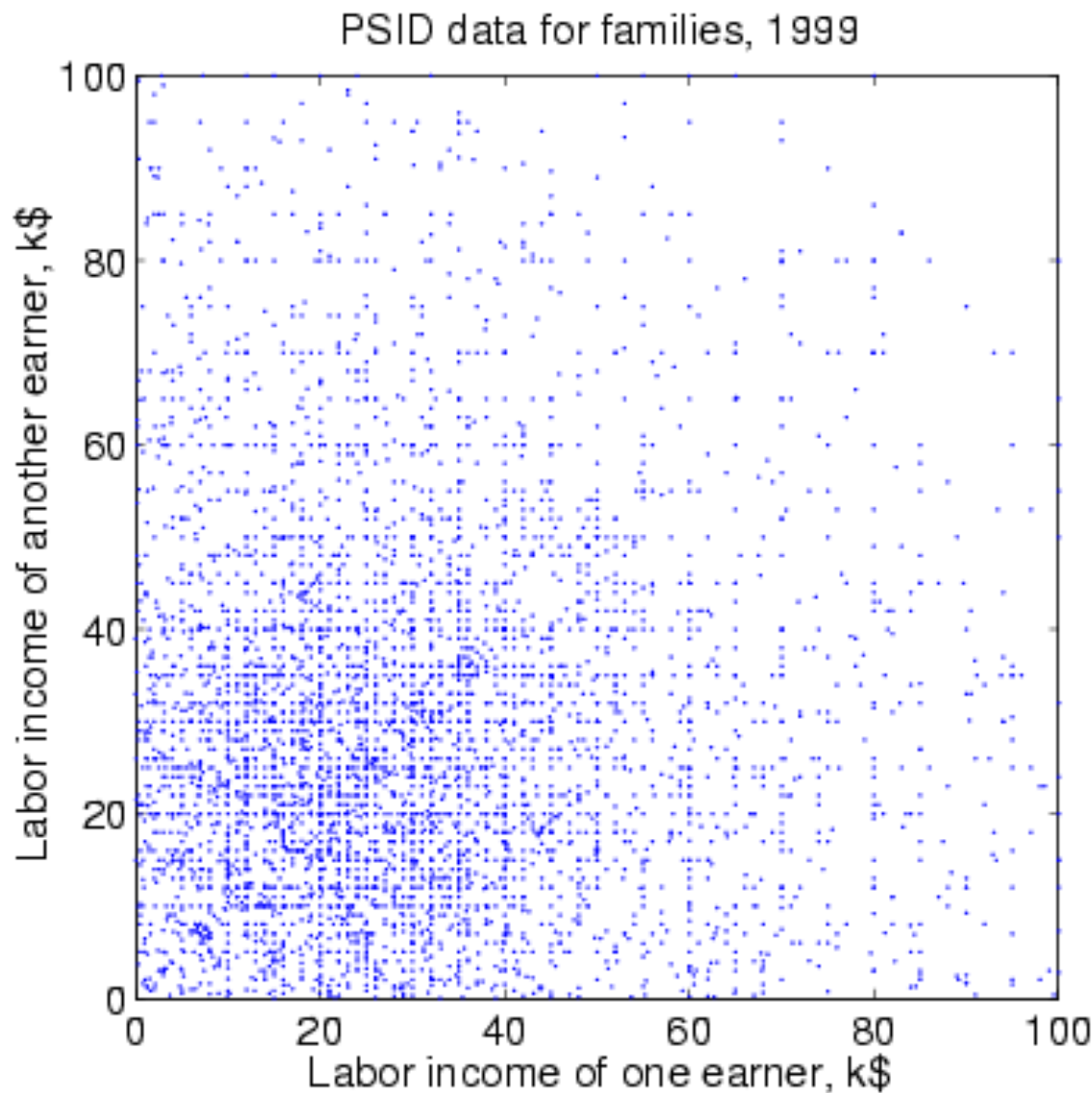


$$r = r' + r'', \quad P_2(r) = \int_0^r P_1(r') P_1(r - r') dr' \propto r \exp(-r / T)$$



The **average** family income is $2T$. The **most probable** family income is T .

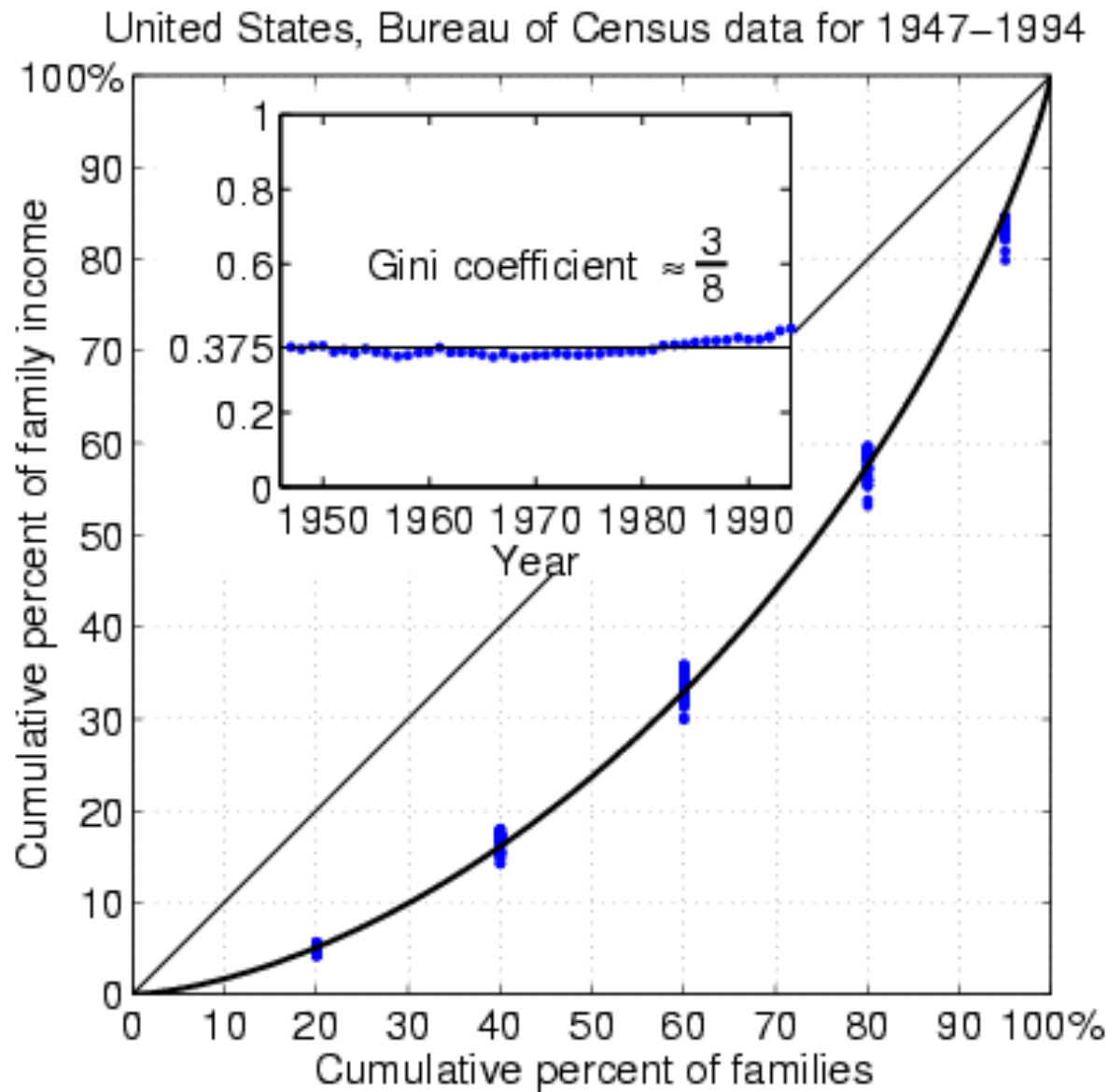
No correlation in the incomes of spouses



Every family is represented by two points (r_1, r_2) and (r_2, r_1) .

The absence of significant clustering of points (along the diagonal) indicates that the incomes r_1 and r_2 are approximately **uncorrelated**.

Lorenz curve and Gini coefficient for families



Lorenz curve is calculated for families $P_2(r) \propto r \exp(-r/T)$. The calculated **Gini coefficient for families** is $G=3/8=37.5\%$

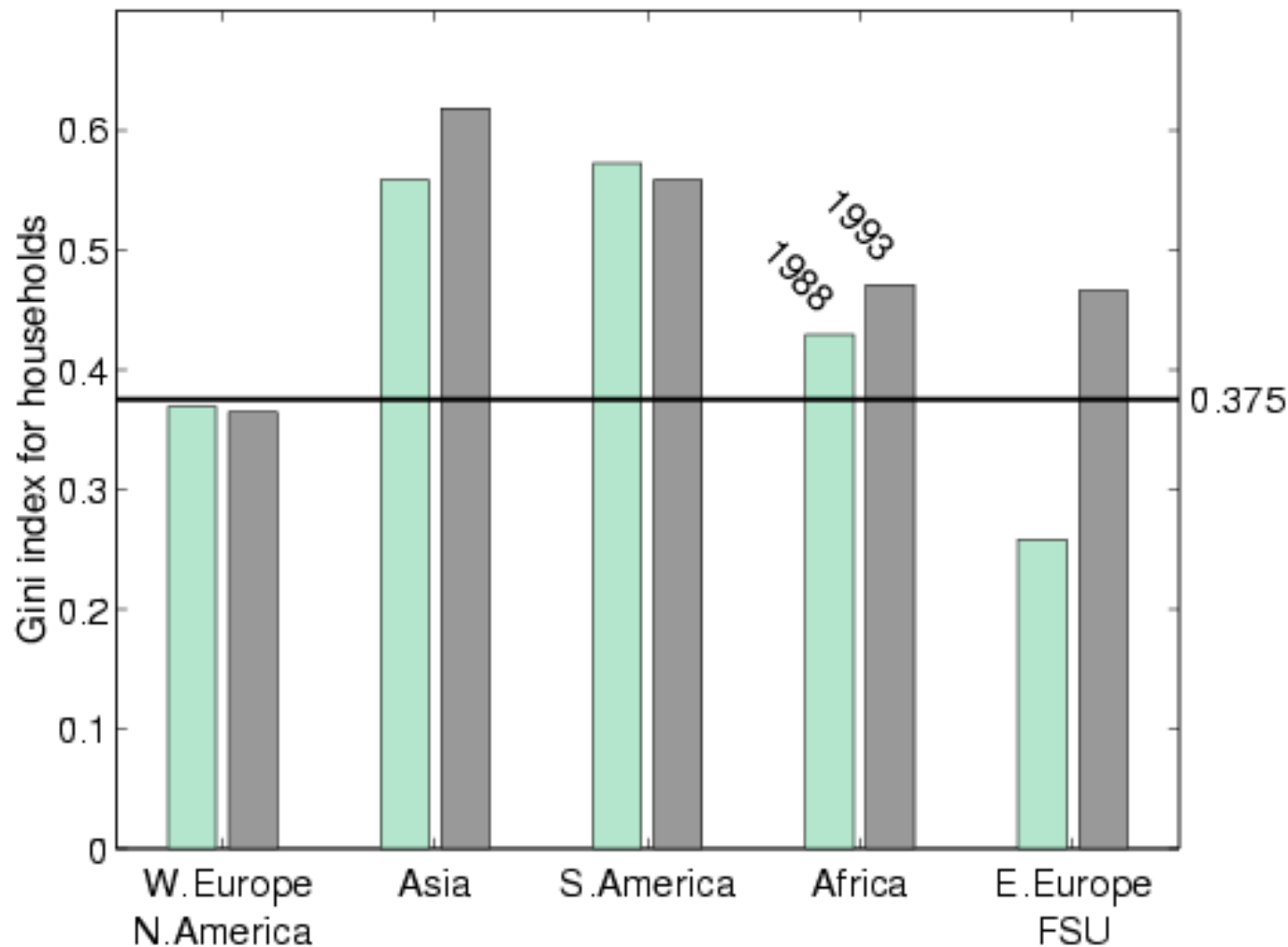
No significant changes in Gini and Lorenz for the last 50 years. The exponential (“thermal”) Boltzmann-Gibbs distribution is very stable, since it maximizes entropy.

Maximum entropy (the 2nd law of thermodynamics) \Rightarrow **equilibrium inequality:**
 $G=1/2$ for individuals,
 $G=3/8$ for families.

World distribution of Gini coefficient

The data from the World Bank (B. Milanović)

World distribution of Gini index, 1988 and 1993



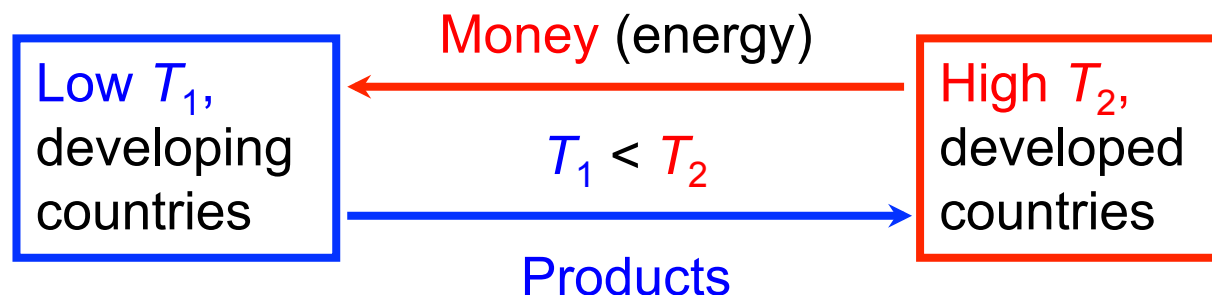
In W. Europe and N. America, G is close to $3/8=37.5\%$, in agreement with our theory.

Other regions have higher G , i.e. higher inequality.

A sharp increase of G is observed in E. Europe and former Soviet Union (FSU) after the collapse of communism – no equilibrium yet.

Thermal machine in the world economy

In general, different countries have different temperatures T , which makes possible to construct a thermal machine:



Prices are commensurate with the income temperature T (the average income) in a country.

Products can be manufactured in a low-temperature country at a low price T_1 and sold to a high-temperature country at a high price T_2 .

The temperature difference $T_2 - T_1$ is the profit of an intermediary.

Money (energy) flows from high T_2 to low T_1 (the 2nd law of thermodynamics – entropy always increases) \Leftrightarrow Trade deficit

In full equilibrium, $T_2 = T_1 \Leftrightarrow$ No profit \Leftrightarrow "Thermal death" of economy