UNIVERSITY OF MARYLAND
Department of Physics
College Park, Maryland

Ph.D. PHYSICS QUALIFYING EXAMINATION - PART I

January 21, 2010 **9 a.m. - 1 p.m.**

Do any four problems. Each problem is worth 25 points.

Put all answers on your answer sheets.

Be sure your Qualifier ID Number is at the top right corner of each sheet and turn in solutions to four problems only. If five solutions are turned in we will grade # 1 - # 4.

Problem I.1

A particle of mass m moves in an isotropic three-dimensional harmonic oscillator potential with natural frequency ω_0. In addition, the particle has electric charge q and moves in crossed applied uniform electric and magnetic fields $\boldsymbol{E} = E_0\hat{\boldsymbol{x}}$ and $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$, where E_0 and B_0 are constants.

(a) Write the non-relativistic Lagrangian for this system. **[5 points]**

(b) Find the stationary position of the particle. **[5 points]**

(c) Obtain the equations of motion for oscillations about this equilibrium. **[5 points]**

(d) From the equations of motion, find the normal modes of oscillations, including their frequencies and normal coordinates. **[5 points]**

(e) Describe the normal modes of oscillations in plain language and qualitatively explain their frequency differences from ω_0. **[5 points]**

Problem I.2

This problem models atomic or molecular scattering of electromagnetic radiation using an idealized classical "atom". An electron is bound to a nucleus located at the origin by a spring constant $k = m\omega_0^2$, where m is the electron mass and ω_0 is the resonance frequency. The "atom" is driven non-relativistically in vacuum by an external electromagnetic plane wave of frequency ω, where $\omega \gg \omega_0$. The plane wave is incident along the y axis and is linearly polarized along x and has amplitude $\tilde{E}$. Assume that the nucleus of the "atom" remains fixed at the origin. Use the notation $f(\boldsymbol{r}, t) = \tilde{f}(\boldsymbol{r})e^{i\omega t}$ for any quantity with simple harmonic time dependence.

(a) Calculate the induced dipole moment $\tilde{\boldsymbol{p}}$ of the driven "atom". **[5 points]**

(b) Then calculate the angular distribution $\langle dP/d\Omega\rangle$ of scattered radiation. You can assume that the radial location of the observation point satisfies $kr \gg 1$, and that $r \gg d_{\text{max}}$, where d_{max} is the maximum spring amplitude. $\langle dP/d\Omega\rangle$ is the cycle averaged power scattered into a solid angle element centered at (θ, ϕ) in spherical coordinates. To start, consider the Poynting vector at $kr \gg 1$. **[5 points]**

(c) From the result in (b), determine the differential scattering cross section $d\sigma/d\Omega$, where $d\sigma/d\Omega = I_0^{-1}\langle dP/d\Omega\rangle$, and I_0 is the incident beam intensity. Then calculate the angularly integrated cross section σ. **[5 points]**

(d) Assuming σ is known, write down an expression describing how the light beam intensity varies along the propagation direction as it scatters in a medium composed of N "atoms" per unit volume. **[5 points]**

(e) Finally, use your answers to the preceding sections to explain why the sky is blue and why sunsets are red. **[5 points]**

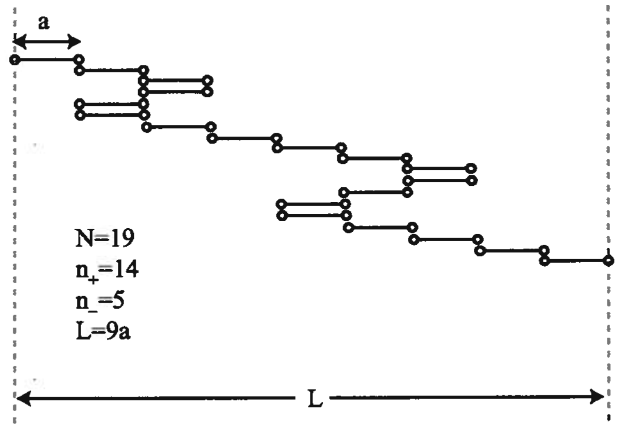
Possibly useful information:

Vector potential (for $kr \gg 1$) at location $\boldsymbol{r} = r\hat{\boldsymbol{r}}$ due to electric dipole $\tilde{\boldsymbol{p}}$ at origin, oscillating at frequency $\omega = ck$:

$$\tilde{\boldsymbol{A}} = -ik\frac{e^{ikr}}{r}\tilde{\boldsymbol{p}} \qquad \text{(Gaussian units), or}$$

$$\tilde{\boldsymbol{A}} = -\frac{i\mu_0\omega}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\tilde{\boldsymbol{p}} \qquad \text{(SI units)}$$

Problem I.3



Consider the following simplified model of polymer elasticity. Assume the polymer forms a 1-dimensional chain consisting of $N \gg 1$ elements, each having length a. Each of the elements in the chain may be freely oriented to the right or to the left, with no energy difference between these two orientations. The extension of the chain is $L = (n_+ - n_-)a$ where n_+ is the number of elements oriented to the right and n_- is the number of elements oriented to the left. See the figure, in which one possible configuration of polymer links is illustrated, where the individual links have been distributed vertically for clarity.

(a) Using a combinatorial definition of the multiplicity W, calculate the entropy of the chain $S = k_B \ln W$ as a function of the extension L for a given N and a. **[7 points]**

(b) From the expression $S(L)$ obtained in part (a), calculate the tension force f necessary to maintain the extension L as a function of the temperature T. **[6 points]**

(c) In the limit of small extension, calculate the effective spring constant for the chain, assuming it can be modeled using Hooke's Law. **[6 points]**

(d) For an unstretched chain ($f = 0$, $\langle L \rangle = 0$) calculate the root mean square fluctuations in the chain extension, $\langle L^2 \rangle^{\frac{1}{2}}$. (Hint: the equipartition theorem may be useful.) **[6 points]**

Problem I.4

In this problem, you will consider the motion of a particle with charge q and relativistic momentum $\boldsymbol{p}$ moving under the influence of a constant electric field $\boldsymbol{E}$. Quantities in ***bold face*** are spatial vectors.

(a) Write down the relativistic formula for the momentum $\boldsymbol{p}$ of an object of rest mass m moving with velocity $\boldsymbol{v}$. **[5 points]**

(b) Assume that the object is initially at rest at the origin at time $t = 0$. Assuming the electric field is $\boldsymbol{E} = E\hat{\boldsymbol{x}}$ with E constant in space and time, determine the object's velocity as a function of time t. **[6 points]**

(c) For the same assumptions as in part (b), determine the object's position as a function of time. **[7 points]**

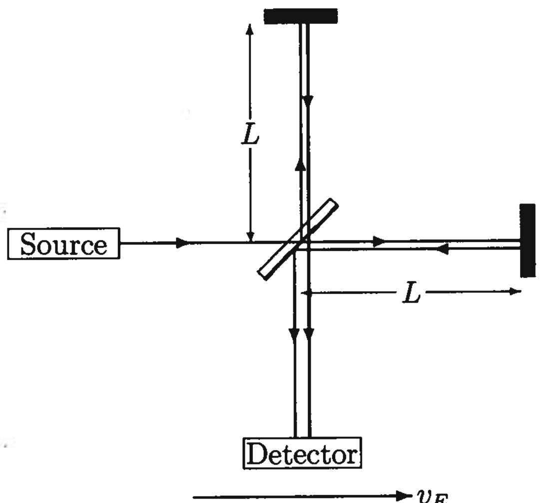
(d) Determine the object's position and velocity at very large and very small times and show that they agree with the expected non-relativistic and ultra-relativistic limits. **[7 points]**

Possibly useful integral:

$$\int \frac{x}{\sqrt{1+bx^2}}\,dx = \frac{\sqrt{1+bx^2}}{b} \;+\; \text{const}$$

Problem I.5

In 1887 Albert Michelson and Edward Morley published the results of their interferometer experiment which was intended to detect the motion of the earth through the ether. In the Michelson apparatus (see Figure), a beam of light is split by a half-silvered mirror, and the two resultant beams travel along perpendicular paths before reflecting off mirrors and being recombined at a detector (a screen or telescope). One of the beams is chosen to travel parallel to the earth's motion relative to the ether, while the other beam travels perpendicular to this direction.



Direction of motion relative to ether

(a) The intensity I of the light at any point on the detector is a function of the phase difference $\Delta\phi$ between the beams arriving at that location. The intensity can be expressed as $I(\Delta\phi) = I_0 f(\Delta\phi)$, where I_0 is the maximum intensity that can be observed, and $f(\Delta\phi)$ is a function which varies between zero and one. Assuming perfect plane waves, and assuming the beams have equal intensity, calculate $f(\Delta\phi)$. You may neglect polarization effects (i.e., you may assume scalar waves). Also note that any phase shifts due to reflection off the mirrors or transmission through the beam splitter are included in $\Delta\phi$, so you need not account for these separately. **[5 points]**

(b) We will analyze the phase difference between the two beams by working in the rest frame of the ether. According to the ether hypothesis, light always travels with speed $c = 3 \times 10^8$ m/s with respect to the fixed ether. Let both interferometer arms have equal length L. Calculate the path length difference Δx of the two beams in terms of L, c, and v_E, the speed of the earth relative to the ether. Keep terms only up to second order in (v_E/c) and ignore any phase shifts due to reflection or beam-splitter effects. **[5 points]**

(c) The effective length of Michelson and Morley's interferometer arm was $L = 12$ meters, and the light source was a sodium lamp of wavelength $\lambda \approx 600$ nm. Define one fringe to be a change in path length difference necessary to make the intensity at the detector go from one maximum to the next maximum. Calculate the number of fringes (or fraction thereof) that should be observed at the detector as the interferometer is rotated by 90 degrees. You may use $v_E = 3 \times 10^4$ m/s. **[5 points]**

(d) Michelson and Morley also used white light as part of their calibration procedure. Explain why a white light source, such as a hot filament or ray of sunlight, might be useful for setting up the apparatus. **[5 points]**

(e) What was the outcome of the Michelson-Morley experiment and what did it prove? **[5 points]**