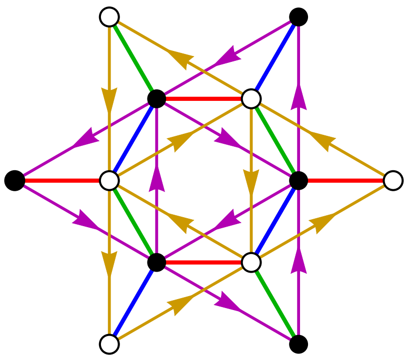
Superconductivity that breaks time-reversal symmetry, and its experimental manifestations



Victor Yakovenko

Condensed Matter Theory Center, Joint Quantum Institute, Dept of Physics, University of Maryland, College Park, USA

- Experimental observation of the polar Kerr effect in the time-reversal symmetry-breaking superconductors
- Relation to the ac Hall conductivity, and the problem of its vanishing for a clean one-band chiral superconductor
- The gauge-invariant time-reversal-odd bilinear product of superconducting pairing terms for a multi-band superconductor
- Loop currents and Hall effect for superconducting pairing on the honeycomb lattice, reminiscent of Haldane's model (1988)

arXiv:1802.02280, *Phys. Rev. X* ***9****, 031025 (2019)*

Philip Brydon, David Abergel, Daniel Agterberg, Victor Yakovenko

Polar Kerr effect and TRSB in Sr_2RuO_4

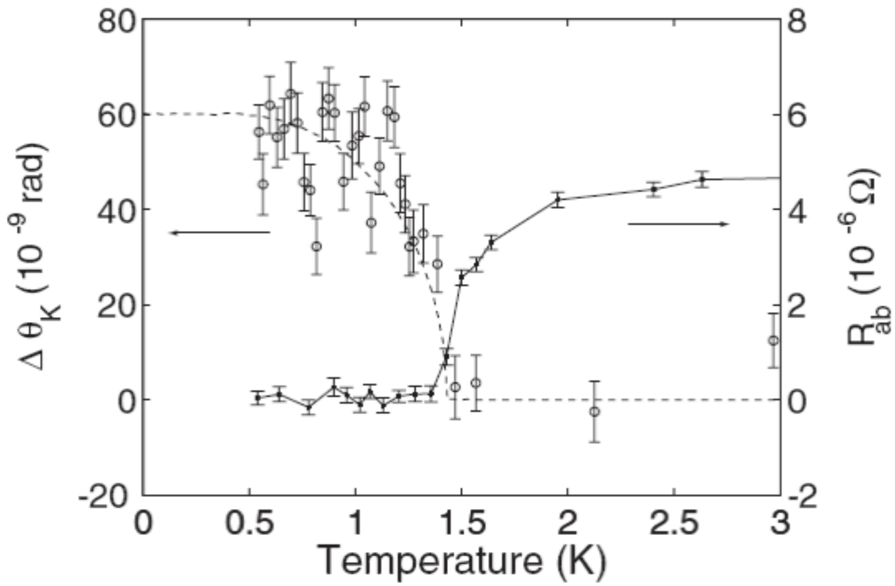


FIG. 2. Zero-field (earth field) measurement of Kerr effect (○) and *ab*-plane electrical resistance (dotted line). Dashed curve is a fit to a BCS gap temperature dependence.

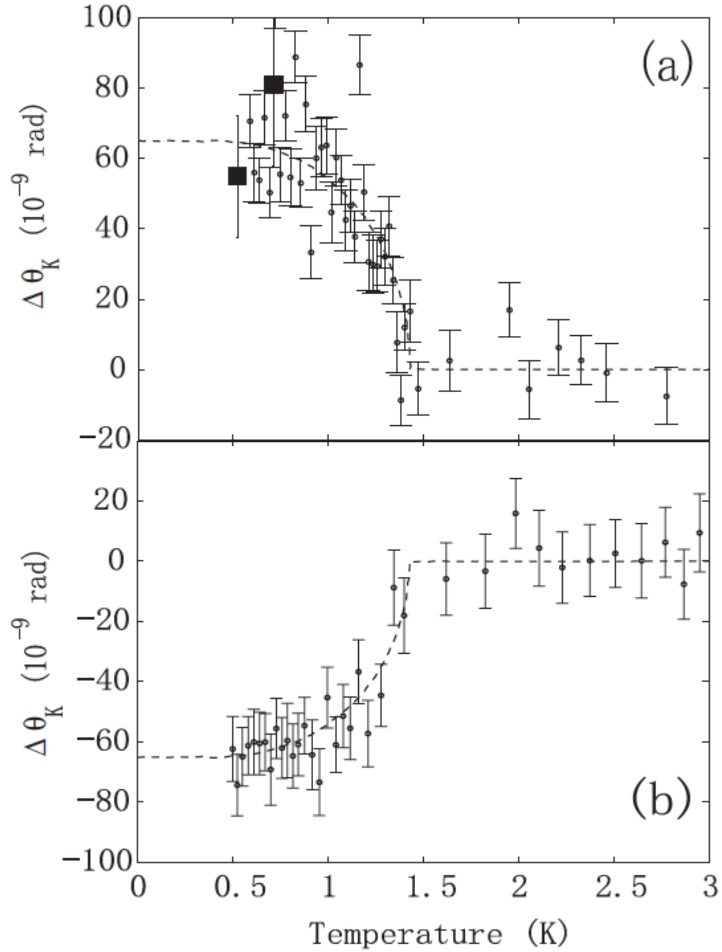
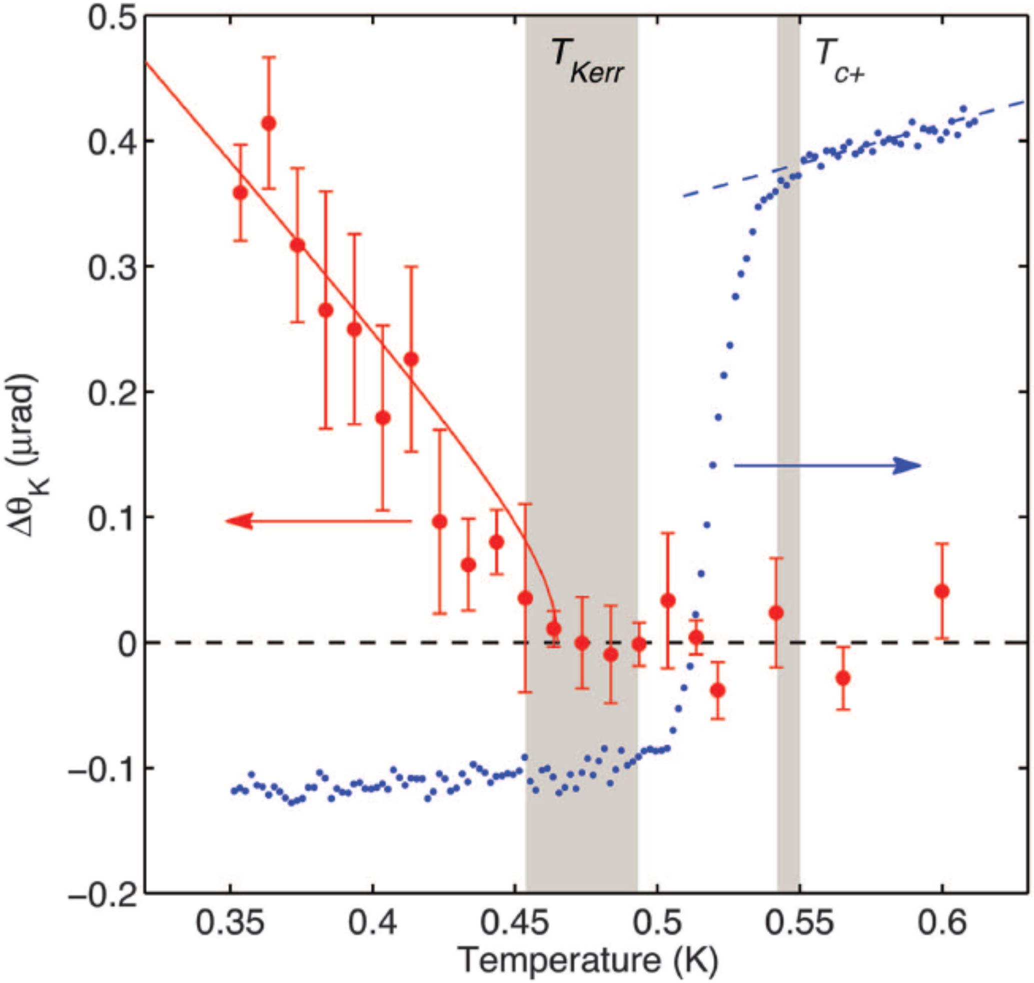
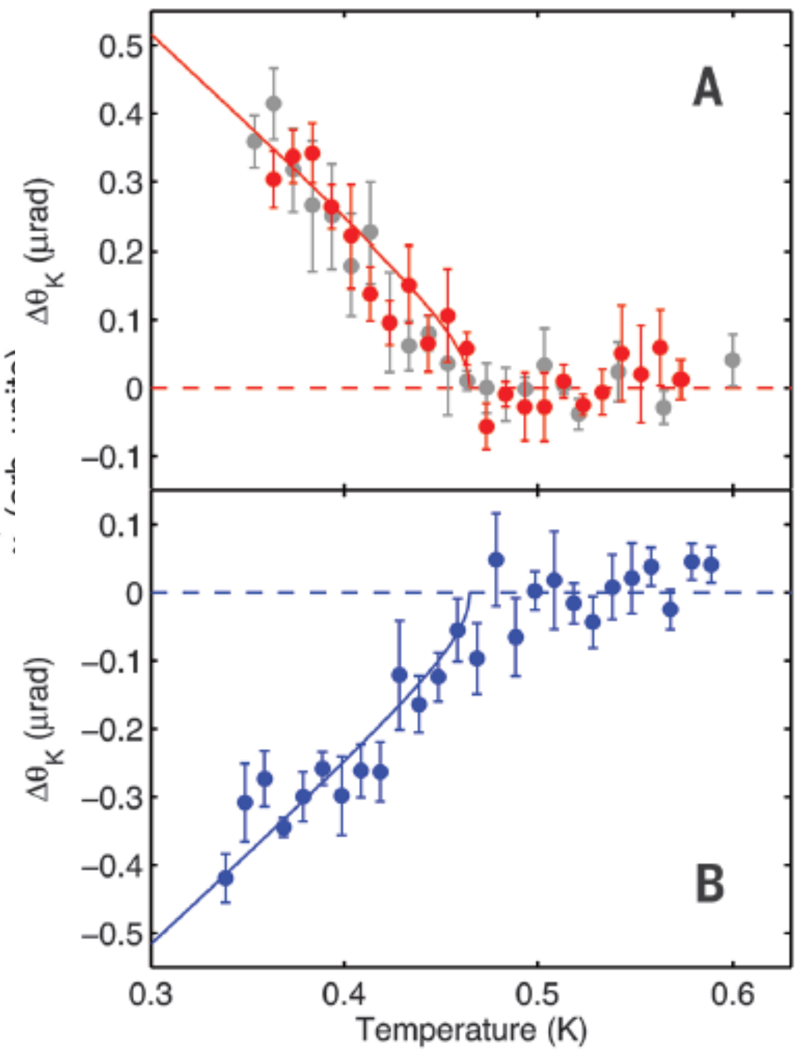


FIG. 3. Representative results of training the chirality with an applied field. (a) +93 Oe field cool, then zero-field warm-up (○). The two solid squares represent the last two points just before the field was turned off. (b) −47 Oe field cool, then zero-field warm-up (○). Dashed curves are fits to a BCS gap temperature dependence.

Xia, …, Kapitulnik,
PRL **97**, 167002 (2006)

Polar Kerr effect and TRSB in UPt_3

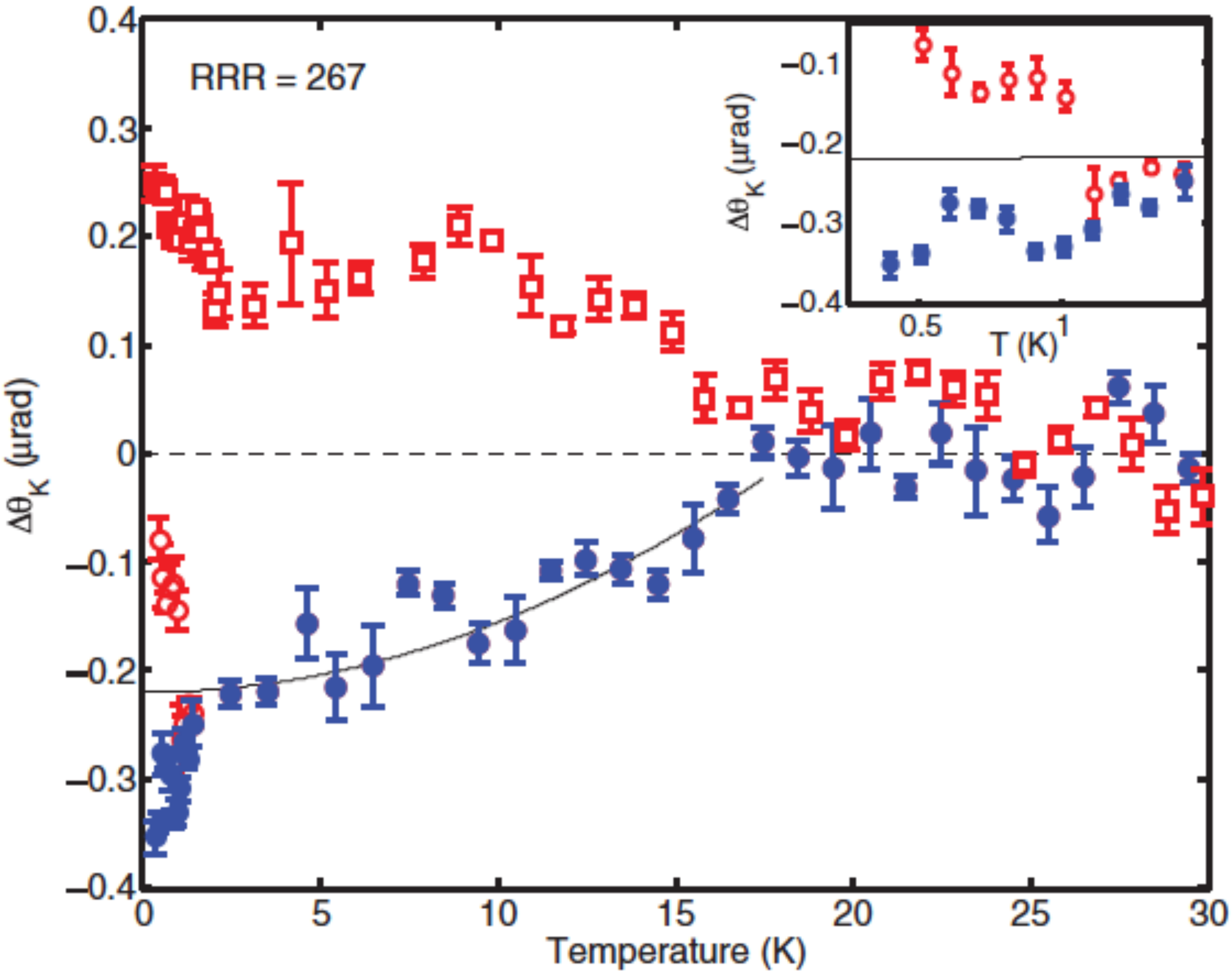




Two superconducting transitions

Schemm, …, Kapitulnik, *Science* **345**, 190 (2014)

Polar Kerr effect and TRSB in URu_2Si_2



Hidden order

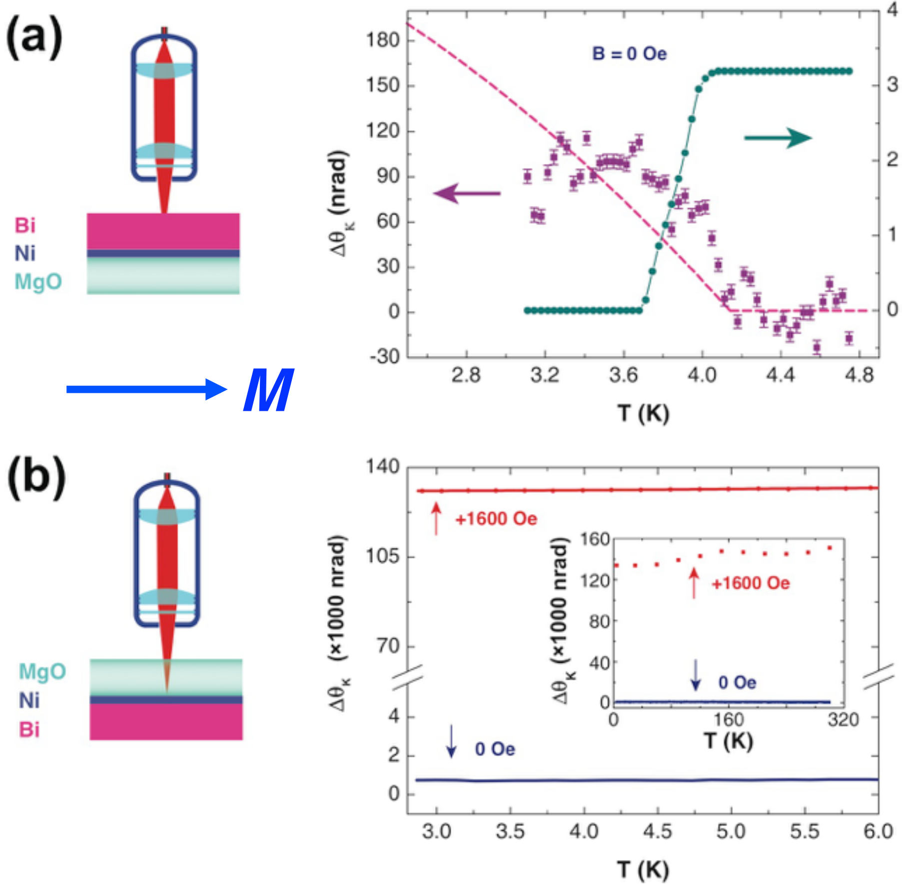
T_{HO}=17.5 K

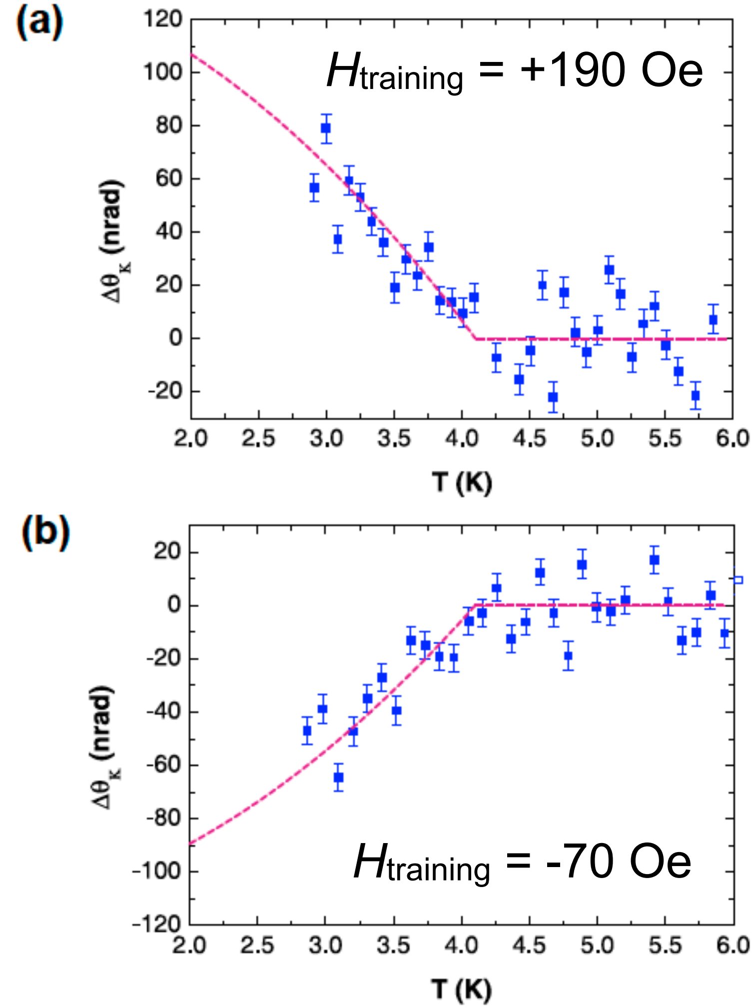
Super-conductivity

T_c=1.5 K

Schemm, …, Kapitulnik, *PRB* **91**, 140506 (2015)

Time-reversal-symmetry-breaking (TRSB) superconductivity in Bi/Ni epitaxial bilayers



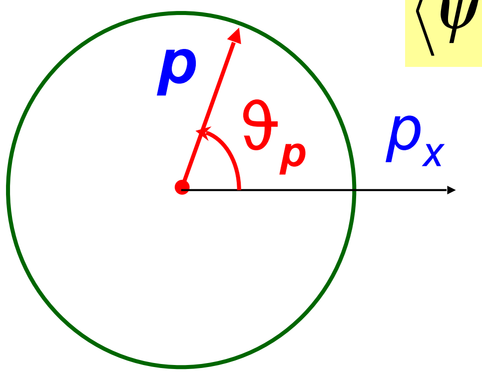


Kerr effect observation in Bi(25 nm)/Ni(2 nm)

Xinxin Gong, Mehdi Kargarian, Alex Stern, Di Yue, Hexin Zhou, Xiaofeng Jin, V.M.Galitski, V.M.Yakovenko, Jing Xia, *Science Advances* 3, e1602579 (2017)

The experiment shows that the superconducting order parameter in Sr_2RuO_4 has intrinsic vorticity. The natural candidate is the triplet p_x+ip_y pairing (Rice & Sigrist 1995, Volovik 1988 2D ^{3}He):

$$\langle \psi(\boldsymbol{p})\psi(-\boldsymbol{p})\rangle \propto \Delta(\boldsymbol{p}) = \Delta_0 \left(p_x + ip_y\right)/p_F = \Delta_0 e^{i\vartheta_{\boldsymbol{p}}}$$



$\Delta(\boldsymbol{p})$ accumulates phase 2π around the Fermi surface – a vortex in momentum space.

It represents the Cooper pairing between electrons with a non-zero angular momentum $L_z=1$.

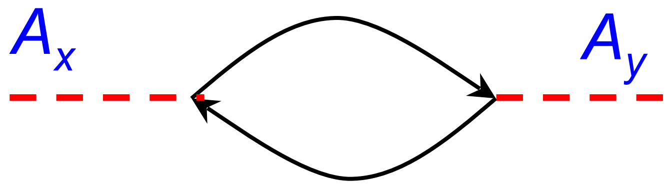
Kerr rotation is permitted by symmetry in this case, but we need to calculate the magnitude of the Kerr angle θ_K.

The experiment gives $\theta_K = 65$ nanorad at $T=0$.

According to the textbook, $\theta_K = \mathrm{Im}\,\dfrac{4\pi\sigma_{xy}(\omega)}{n\left(n^2-1\right)\omega d}$

where n is the refraction coefficient, d is the interlayer distance.

The ac Hall conductivity $\sigma_{xy}(\omega)$ can be obtained by calculating the one-loop current-current response function

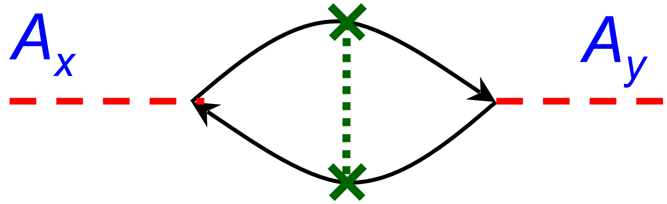


using the Nambu Green function for a chiral superconductor

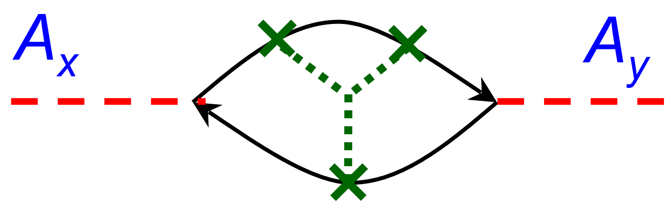
$$G = -\frac{i\omega + \varepsilon(\boldsymbol{p})\hat{\tau}_3 + p_x\Delta_x\hat{\tau}_1 - \Delta_y p_y\hat{\tau}_2}{\omega^2 + E^2(\boldsymbol{p})}$$

However, this diagram vanishes identically (by taking traces, before integration), even though it is permitted by symmetry.

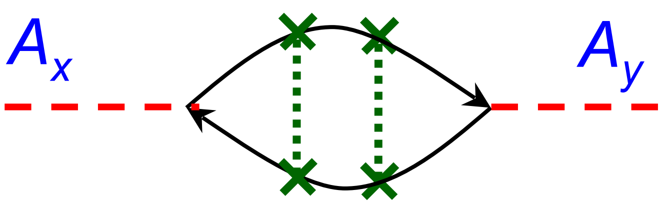
Impurity scattering may produce non-zero Hall effect, although the lowest-order diagram vanishes identically (by taking traces):

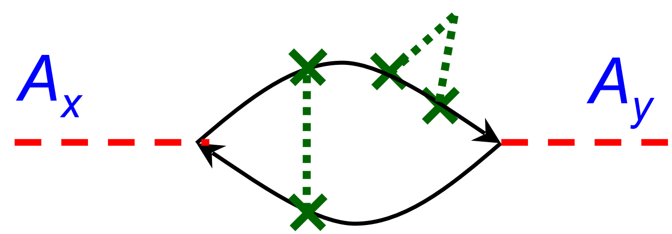


A non-zero Hall effect comes from the following diagrams:



Goryo PRB 2008: In this diagram, the sign of the Hall effect is determined by the sign of the impurity potential.

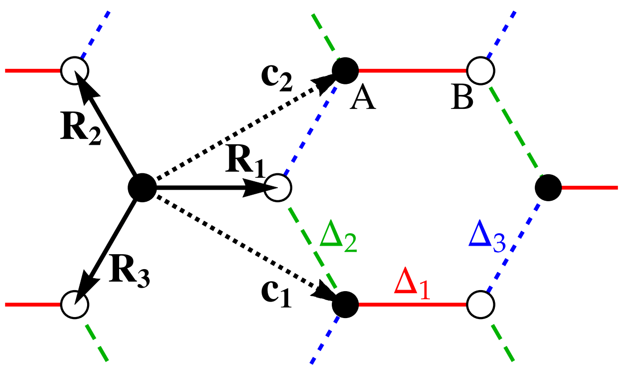




Lutchyn, Nagornykh, Yakovenko, *PRB* **80**, 104508 (2009): The Hall effect sign depends on the electron-hole asymmetry.

Anomalous $\sigma_{xy}(\omega)$ for a multi-band superconductor

- Taylor, Kallin, *PRL* **108**, 157001 (2012) 3-band Sr_2RuO_4; Wang, Berlinsky, Zwicknagl, Kallin, *PRB* **96**, 174511 (2017) UPt_3
- Gradhand, Wysokiński, Annett, Györffy, *PRL* **108**, 077004 (2012); *PRB* **88**, 094504 (2013); *J Phys Cond Mat* **26** 274205 (2014); *Phil Mag* **95**, 525 (2015)
- Brydon, Abergel, Agterberg, Yakovenko, arXiv:1802.02280, Two-band superconducting pairing on the honeycomb lattice: the simplest, minimal conceptual model

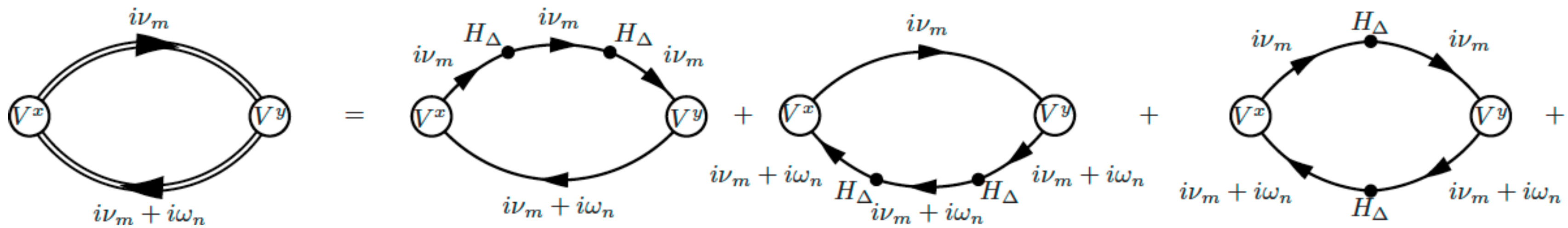


$$\Delta_\pm(\mathbf{k}) = \Delta^x_{\mathbf{k}} s_x + \Delta^y_{\mathbf{k}} s_y = \Delta \sum_{j=1}^{3} e^{\mp i\phi_j} \begin{pmatrix} 0 & e^{i\mathbf{k}\cdot\mathbf{R}_j} \\ e^{-i\mathbf{k}\cdot\mathbf{R}_j} & 0 \end{pmatrix}$$

$$(\Delta_1, \Delta_2, \Delta_3) = \Delta(1, e^{\pm 4\pi i/3}, e^{\pm 8\pi i/3})$$

matrix order parameter for singlet *d*-wave chiral pairing

Time-reversal-odd bilinear (TROB) product



Observables depend on gauge-invariant bilinear products of Δ and Δ^+. A time-reversal-odd bilinear (TROB) product is needed for time-reversal-odd observations, e.g. Hall effect:

$$\text{TROB} = \Delta(\mathbf{k})\Delta^\dagger(\mathbf{k}) - \Delta^\dagger(\mathbf{k})\Delta(\mathbf{k}) = [\Delta(\mathbf{k}), \Delta^\dagger(\mathbf{k})]$$

Obviously, TROB=0 for a single-band scalar Δ.

For a two-band model, we expand over 2x2 Pauli matrices:

$$\Delta(\mathbf{k}) = \Delta_0(\mathbf{k})s_0 + \mathbf{\Delta}(\mathbf{k})\cdot\mathbf{s} \quad \Rightarrow \quad [\Delta(\mathbf{k}), \Delta^\dagger(\mathbf{k})] = 2i[\mathbf{\Delta}(\mathbf{k}) \times \mathbf{\Delta}^*(\mathbf{k})]\cdot\mathbf{s}$$

TROB≠0 only for a non-unitary pairing with Δ not parallel to Δ^*

TROB=0 for perfect superconducting fitness, when the normal-state Hamiltonian $H_0(\mathbf{k})$ commutes with $\mathbf{\Delta}$(k) and both can be diagonalized

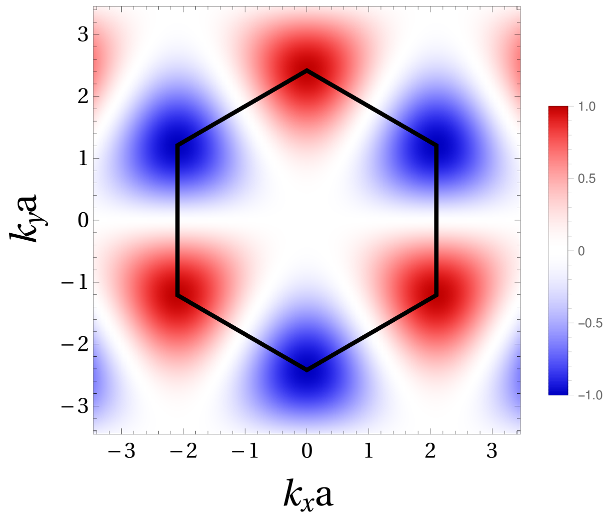
$$H_0(\mathbf{k}) = h_0(\mathbf{k})s_0 + \mathbf{h}(\mathbf{k})\cdot\mathbf{s} \quad \Rightarrow \quad [H_0(\mathbf{k}), \Delta(\mathbf{k})] = 2i[\mathbf{h}(\mathbf{k}) \times \mathbf{\Delta}(\mathbf{k})]\cdot\mathbf{s}$$

TROB for the honeycomb lattice model

For the honeycomb lattice model, TROB is expressed in terms of the sublattice polarization $\Xi_\pm(\mathbf{k}) = \mathrm{Tr}\{\Delta_\pm^\dagger(\mathbf{k}) s_z \Delta_\pm(\mathbf{k})\}$

$$\Delta_\pm(\mathbf{k}) = \Delta_\mathbf{k}^x s_x + \Delta_\mathbf{k}^y s_y$$

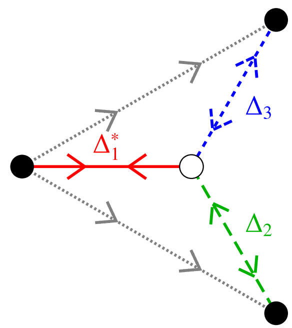
$$\mathrm{TROB} = \mp 2 s_z \mathrm{Im}[\boldsymbol{\Delta}_\mathbf{k} \wedge \boldsymbol{\Delta}_\mathbf{k}^*] = s_z \Xi_\pm(\mathbf{k})$$



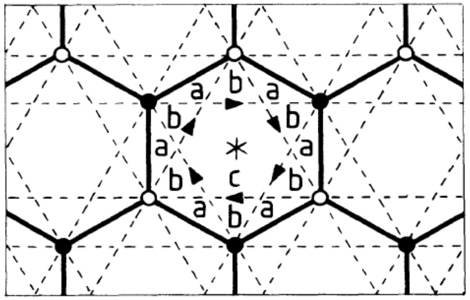
We will see below that the TROB determines the existence of loop currents, Hall conductivity, and other manifestations of time-reversal symmetry breaking in the honeycomb model.

Loop currents on the honeycomb lattice

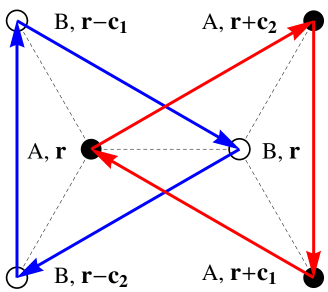
The nearest-neighbor superconducting pairing generates next-nearest-neighbor tunneling with complex amplitudes



Haldane, *PRL* (1988)

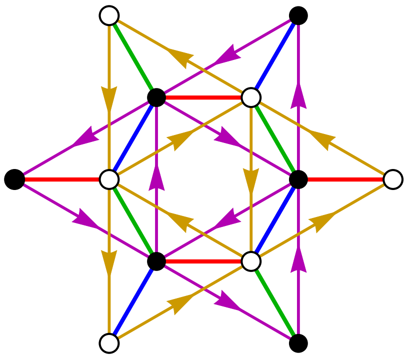


The complex next-nearest-neighbor tunneling amplitudes imply the existence of loop currents around each lattice site.



$$\chi = i \sum_{\mathbf{r},\sigma} \Big(a^\dagger_{\mathbf{r},\sigma} a_{\mathbf{r}+\mathbf{c}_1,\sigma} + a^\dagger_{\mathbf{r}+\mathbf{c}_1,\sigma} a_{\mathbf{r}+\mathbf{c}_2,\sigma} + a^\dagger_{\mathbf{r}+\mathbf{c}_2,\sigma} a_{\mathbf{r},\sigma}$$

$$+ b^\dagger_{\mathbf{r},\sigma} b_{\mathbf{r}-\mathbf{c}_1,\sigma} + b^\dagger_{\mathbf{r}-\mathbf{c}_1,\sigma} b_{\mathbf{r}-\mathbf{c}_2,\sigma} + b^\dagger_{\mathbf{r}-\mathbf{c}_2,\sigma} b_{\mathbf{r},\sigma} - \text{H.c.} \Big)$$



The loop current operator χ has a non-zero expectation value proportional to TROB in the superconducting state:

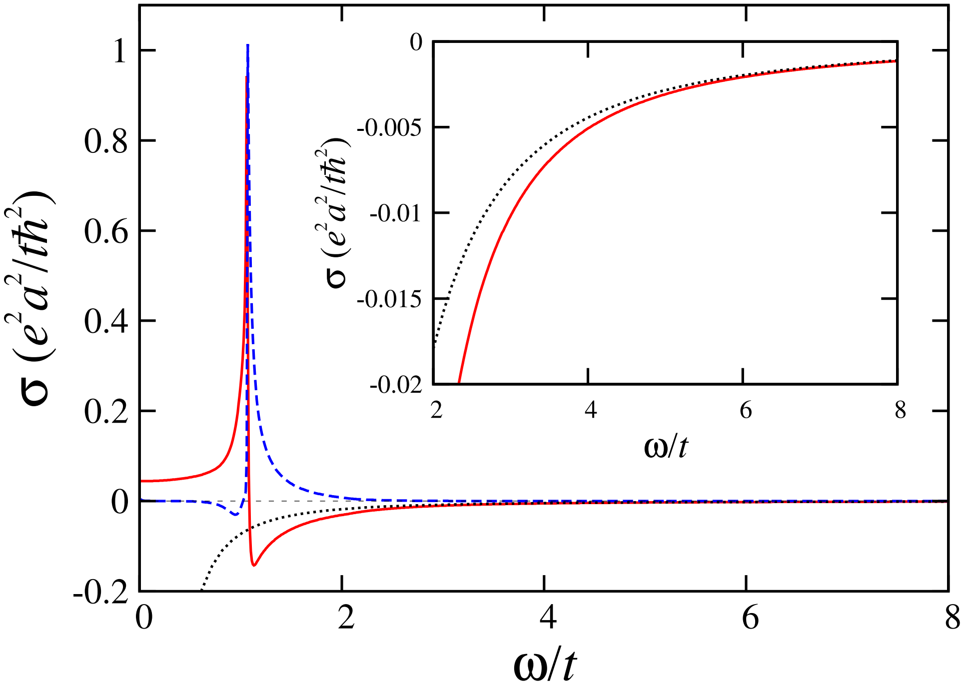
$$\frac{\langle \chi \rangle}{N} = -\frac{1}{N} \sum_{\mathbf{k}} \frac{1}{\beta} \sum_{\nu_m} \sin\!\left(\frac{\sqrt{3}}{2} k_y a\right) \left[\cos\!\left(\tfrac{3}{2} k_x a\right) - \cos\!\left(\frac{\sqrt{3}}{2} k_y a\right) \right] \frac{8\mu\, \mathrm{Tr}\{\Delta^\dagger(\mathbf{k}) s_z \Delta(\mathbf{k})\}}{(\nu_m^2 + E^2_{\mathbf{k},1})(\nu_m^2 + E^2_{\mathbf{k},2})}$$

Hall conductivity in the honeycomb model

Hall conductivity is obtained from a correlator of current operators:

$$\mathbf{J} = e\sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}}\,\tau_0 \otimes \begin{pmatrix} 0 & \mathbf{v}_{\mathbf{k}} \\ \mathbf{v}^*_{\mathbf{k}} & 0 \end{pmatrix} \Psi_{\mathbf{k}}$$

$$\sigma_H(\omega) = \lim_{i\omega_n \to \omega + i0^+} \frac{e^2}{N\beta} \sum_{\mathbf{k},\nu_m} \frac{\mu\; i[\mathbf{v}^*_{\mathbf{k}} \wedge \mathbf{v}_{\mathbf{k}}]\;\mathrm{Tr}\{\Delta^\dagger(\mathbf{k}) s_z \Delta(\mathbf{k})\}\;(i\omega_n + 2i\nu_m)^2}{(\nu_m^2 + E^2_{\mathbf{k},1})(\nu_m^2 + E^2_{\mathbf{k},2})([\omega_n+\nu_m]^2 + E^2_{\mathbf{k},1})([\omega_n+\nu_m]^2 + E^2_{\mathbf{k},2})}$$



In high-frequency limit, Hall conductivity is proportional to the expectation value of the loop current operator **χ**

$$\sigma_H(\omega) \approx \frac{i\,\langle [J_x, J_y]\rangle}{N\omega^2} = -\frac{\sqrt{3}e^2t^2a^2}{2N\omega^2\hbar^2}\langle\chi\rangle$$

Some similarity to the Berry curvature method in Haldane's model

Separate transitions for Z_2 and U(1) symmetry breaking?

The bilinear time-reversal-odd product $[\Delta,\Delta^+]$ represents spontaneous breaking of the discrete Z_2 time-reversal symmetry, e.g. by clockwise or counterclockwise loop currents.

The linear pairing potential Δ represents spontaneous breaking of the continuous U(1) gauge symmetry, resulting in superfluidity.

Spontaneous breaking of these two symmetries may happen at two separate transitions: $<[\Delta,\Delta^+]>\neq 0$ at T_{TRSB} and $<\Delta>\neq 0$ at T_c

It is plausible that $T_c < T_{TRSB}$, so $<\Delta>=0$ due to phase fluctuations in the range $T_c < T < T_{TRSB}$, but $<[\Delta,\Delta^+]>\neq 0$ and Z_2 is broken.

The two transitions can be distinguished experimentally:

	Polar Kerr effect	Supercurrent
$T_c<T_{TRSB}<T$	No	No
$T_c<T<T_{TRSB}$	Yes	No
$T<T_c<T_{TRSB}$	Yes	Yes

Also discussed by Yegor Babaev, Andrey Chubukov, …

Conclusions [arXiv:1802.02280, PRX 2019]

- The polar Kerr effect observed in the time-reversal symmetry-breaking superconductors is related to the ac Hall conductivity.
- The Hall effect for a clean one-band chiral superconductor (without impurities) vanishes identically.
- A non-zero Hall effect can be obtained only if impurity scattering or multi-band superconductivity is taken into account.
- A non-zero time-reversal-odd bilinear (TROB) combination of superconducting pairing is necessary for a non-zero Hall effect.
- The nearest-neighbour chiral *d*- or *p*-wave pairing on the honeycomb lattice has a non-zero TROB, manifested as the sublattice polarization of pairing.
- As in Haldane's model, we find persistent loop currents around each site and non-zero Hall conductivity, both expressed in terms of TROB and resulting in the polar Kerr effect.
- Discrete Z_2 time-reversal symmetry and continuous U(1) gauge symmetry may be broken at different temperatures.