(Mostly) Quantum Cosmology

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The part that's not quantum cosmology

Some fun with physics

Taub-NUT Space as a Counterexample to Almost Anything

by

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Misner space as a prototype for almost any pathology

K.S. Thorne (Caltech, Kellogg Lab)
1993

14 pages
Contribution to: Directions in General Relativity: An International Symposium in Honor of the 60th Birthdays of Dieter Brill and Charles Misner, 333-346

PHYSICAL REVIEW VOLUME 116, NUMBER 4 NOVEMBER 15, 1959

Active Gravitational Mass*

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(Received June 22, 1959)

Tolman states that ". . .disordered radiation in the interior of a fluid sphere contributes roughly speaking twice as much to the gravitational field of the sphere as the same amount of energy in the form of matter." The gravitational pull exerted by a system on a distant test particle might therefore at first sight be expected to increase if within the system a pair of oppositely charged electrons annihilate to produce radiation. This apparent paradox is analyzed here in the case where gravitational effects internal to the system are unimportant. It is shown that tensions in the wall of the container compensate the effect mentioned by Tolman so that the net gravitational pull exerted by the system does not change.

I. INTRODUCTION

IN Newtonian mechanics the equivalence of active and passive gravitational mass, that is of mass as a quantity which gives rise to, and as a quantity acted upon by, gravitational fields, is made obvious in the form of the familiar equation for the gravitational potential ϕ, namely $\nabla\phi=4\pi\rho$, where ρ is the density of inertial mass.

However, in relativity theory where the field equations take the form $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=T_{\mu\nu}$, the inference can sometimes not be drawn so easily. Here not only does the source term include stresses and momenta as well as energy, but the equations are nonlinear. The question presents itself, therefore, to what extent are the distant gravitational fields as calculated by classical and special relativity theory the same as those calculated using general relativity?

The following statement by Tolman suggests that there are important differences: ". . .disordered radiation in the interior of a fluid sphere contributes roughly speaking twice as much to the gravitational field of the sphere as the same amount of energy in the form of matter."[1]

Such a result would seem to lead to certain paradoxes. Consider the conversion of a gamma ray, enclosed in a box, into mass, say an electron-positron pair. This transformation might be thought to halve the contribution of the mass energy to distant gravitational fields.

However, we shall show here that the active gravitational mass of a system is made up of the energy of the walls and other material plus the energy of radiation, divided by c^2, without the added factor of two, provided that the gravitational fields internal to the system are weak.

II. ENCLOSED RADIATION

Tolman's argument is based upon an expression for the distant gravitational field which involves only the classical stress-energy tensor $T_{\mu\nu}$. The reasoning applies to a wide class of cases roughly describable as quasi-static. Included in such cases are those in which the matter is confined to some limited region. This region is considered to be small as compared to the distance at which its gravitational field is to be measured. Moreover, within this region the behavior of the system is not significantly influenced by its own gravitational field. When these conditions are satisfied, and when the distant metric field is expressed in a form,

$$ds^2=-(1+2m^*/r)(dx^2+dy^2+dz^2)+(1-2m^*/r)dt^2, \quad (1)$$

which reveals the mass of the system, $m=(c^2/G)m^*$, or its energy $E=mc^2=(c^4/G)m^*$, then Tolman's arguments[2] give for the energy of the system the value

$$mc^2=(c^4/G)m^*=\int(T_4{}^4-T_1{}^1-T_2{}^2-T_3{}^3)(-g)^{\frac{1}{2}}d^3x. \quad (2)$$

Since the electromagnetic stress-energy tensor has zero trace, it follows that $T_4{}^4$ equals $-(T_1{}^1+T_2{}^2+T_3{}^3)$. Therefore according to (2), Tolman argues, the system

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† Fellow of the Alfred P. Sloan Foundation during a part of the period of this work.

[1] R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Clarendon Press, Oxford, 1934), p. 272.

[2] See Tolman, reference 1, p. 235, Eq. (92.3).

light bends twice as much as matter

light contributes $\rho+3p$ in cosmology

so. . .

does light weigh twice as much as matter?

The part that is quantum cosmology

1915: Einstein publishes the general theory of relativity

1916: Einstein suggests gravity must be combined with quantum theory

1925: Schrödinger equation, Heisenberg's matrix mechanics

1930: first serious attempt to quantize general relativity (Rosenfeld)

⋮

2023: still not there

Why is this so hard?

REVIEWS OF MODERN PHYSICS VOLUME 29, NUMBER 3 JULY, 1957

Feynman Quantization of General Relativity*†

CHARLES W. MISNER

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- general relativity is "topologically invariant" (diffeomorphism invariant)
- Hamiltonian is identically zero

  Schrödinger equation $i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle = 0$ — "frozen time"

  "In any topologically invariant theory, the Hamiltonian operator vanishes."
- physical observables are nonlocal

  "coordinates" are not the same as "events"

  $\mathcal{O}(x)$ depends on coordinate x; true observables depend on events
- without time, it becomes hard to define "spacelike" or "causal"
- vacuum state probably "foamy" at small distances

Four approaches to quantization:

- Perturbation theory around flat spacetime
- Canonical quantization
- Feynman path integral
- Schwinger variational principle

Some key technical issues:

- Importance of constraints, gauge fixing
- Path integral measure
- Need for (and difficulty of finding) inner product

Full Hamiltonian constraint:

$$\left(16\pi{\ell_p}^2 G_{ijkl}\frac{\delta}{\delta q_{ij}}\frac{\delta}{\delta q_{kl}} + \frac{1}{16\pi{\ell_p}^2}\sqrt{q}\,{}^{(3)}R\right)\Psi[q] = 0$$

$$^{(3)}\nabla_i\left(\frac{\delta\Psi[q]}{\delta q_{ij}}\right) = 0$$

Too hard!

Minisuperspace: freeze out all but a few degrees of freedom

Which degrees of freedom do you keep?

DeWitt (1967): quantize homogeneous isotropic universe

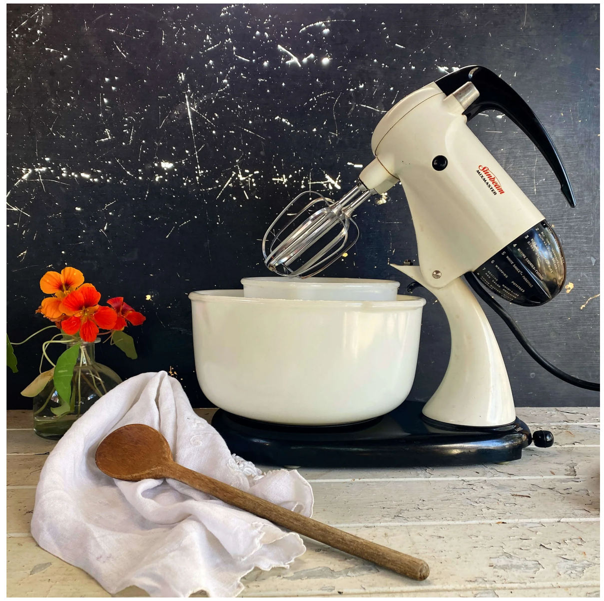
Vacuum behavior: uniform expansion, no dynamical degrees of freedom

Misner (1968): Kasner Universe: anisotropic expansion

$$ds^2 = dt^2 - l_1^2(t)dx^2 - l_2^2(t)dy^2 - l_3^2(t)dz^2$$

Vacuum behavior: expansion along two axes, contraction along the third

Misner (1969): Mixmaster Universe



$$
\begin{aligned}
ds^2 = {} & N^2(t)dt^2 \\
& - e^{2(-\Omega+\beta_+ +\sqrt{3}\beta_-)}(\sin\psi d\theta - \cos\psi \sin\theta d\phi)^2 \\
& - e^{2(-\Omega+\beta_+ -\sqrt{3}\beta_-)}(\cos\psi d\theta + \sin\psi \sin\theta d\phi)^2 \\
& - e^{2(-\Omega-2\beta_+)}(-d\psi - \cos\theta d\phi)^2
\end{aligned}
$$

Vacuum behavior:

- like Kasner with “bounces,” abrupt changes in axes and rates
- can be viewed as including longest wavelength gravitational waves

Mixmaster animated:

https://upload.wikimedia.org/wikipedia/commons/1/17/BKLChaotic.gif

(Lantonov, Wikimedia Commons)

Belinski, Khalatnikov, Lifshitz:

locally Mixmaster behavior near generic spacelike singularity

PHYSICAL REVIEW VOLUME 186, NUMBER 5 25 OCTOBER 1969

Quantum Cosmology. I*†

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(Received 8 July 1969)

The Hamiltonian methods of Arnowitt, Deser, and Misner can be applied to homogeneous cosmological models, and prove to be an efficient way both of constructing the Einstein equations and of studying their solutions. By using an appropriate form for the metric, one finds that the constraint equations for these models can be solved explicitly, and the resulting problem in Hamiltonian mechanics resembles that of a particle in a potential well. The most unusual feature of the Hamiltonian is that it is explicitly time dependent. There is an easy and attractive choice of factor orderings which allows one to pass on to a quantum theory (by imposing canonical commutation relations on the independent canonical variables) while maintaining the signature of the quantized metric. For the closed-space cosmological model (Bianchi type IX) which is studied in most detail, a classical (high-quantum-number) state remains classical as the wave function is followed back in time toward the initial singularity. There is no tendency for significant contributions from states of low quantum number to develop even when the radius of the universe is much less than $(G\hbar/c^3)^{\frac{1}{2}}=10^{-33}$ cm.

- time: use spatial volume (or Ω) as time coordinate
  (but different from usual cosmological time)
- observables: “shape parameters” $\beta_\pm$ and their derivatives
  (these are nonlocal, invariant)
- Hamiltonian has discrete spectrum
- (for the experts: reduced phase space quantization)

Basic question:

does quantum gravity cure the initial big bang singularity?

Misner's answer: probably not;

nearly classical state will remain nearly classical near zero volume

Later:

- answer may depend on choice of variables or commutators
- answer may depend on choice of time
- answer may depend on how you define a quantum singularity

Continuing program (451 citations to *Quantum Cosmology* so far. . .)

Misner (1973): Gowdy $\boldsymbol{T^3}$ cosmology

- gravitational waves in an expanding closed universe
- “midisuperspace”: infinite number of degrees of freedom
- problem of choosing initial conditions
- (for the experts: mixed reduced phase space and Dirac quantization)

Charlie Misner

- established the basic conceptual issues of quantum gravity (in 1957!)
- developed a profound program to start addressing them
- largely founded quantum cosmology
- . . . and clearly had enormous fun along the way