Black Hole Perturbations, Ringdowns, and All That

Saul Teukolsky
Cornell University & Caltech

Misner Symposium, Nov 11, 2023

A Little History . . .

- Weber (1969)
- Misner:
  - Beaming
  - Gravitational synchrotron radiation
  - Superradiant scattering

Interpretation of Gravitational-Wave Observations*

C. W. Misner
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 22 November 1971; revised manuscript received 13 March 1972)

If Weber's gravitational-wave observations are interpreted in terms of a source at the Galactic center, both the intensity and the frequency of the waves are more reasonable if the source is assumed to emit in a synchrotron mode (narrow angles, high harmonics). Although presently studied sources for such modes are astrophysically unsatisfactory —high-energy, nearly circular, scattering orbits—other possible sources are under study.

Gravitational Synchrotron Radiation in the Schwarzschild Geometry*

C. W. Misner, R. A. Breuer,† D. R. Brill, P. L. Chrzanowski, H. G. Hughes, III, and C. M. Pereira
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 9 December 1971)

The existence of a mechanism for gravitational synchrotron radiation is demonstrated in solutions of the wave equation in the Schwarzschild background, with the source a particle in a highly relativistic circular geodesic. The main features (high-frequency harmonics, narrow angular distribution in latitude) are shown to hold for vector (electromagnetic) and tensor (gravitational) radiations, which are expected to be strongly polarized in the orbit plane. Detailed formulas for the spectrum are given in the scalar case.

Black Hole Perturbations. . .

- Regge & Wheeler (1957), Zerilli (1970)

$$\frac{d^2 R}{d r_*^2} + \left[\omega^2 - V(r)\right] R = 0$$

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- What about Kerr?

Kip's grad students (1971)

Jim Ipser, Richard Price, Bernie Schutz, Cliff Will, Bill Press . . .

∼ 1974

NP and all that

$$F_{ab} \leftrightarrow \{\Phi_0, \Phi_1, \Phi_2\}$$

$$C_{abcd} \leftrightarrow \{\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4\}$$

$$\Psi_4 \sim \frac{d^2h}{dt^2}$$

- Price: R-W with $\mathrm{Im}(\Psi_2)$
- Fackerell & Ipser: Decoupled eqn for Φ_1 (Kerr)
- Bardeen & Press: Eqns for Ψ_0, Ψ_4 (Schw)
- “If it works for Schw, it’ll work for Kerr”

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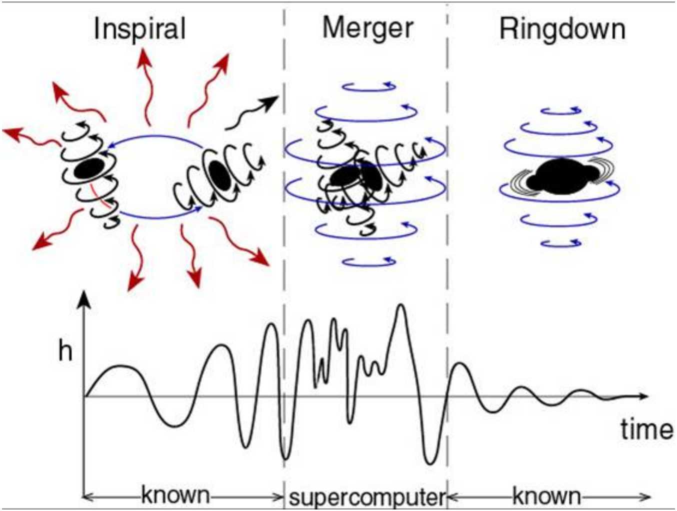
- Turns out not Charlie's fault . . .

6 Months Later . . .

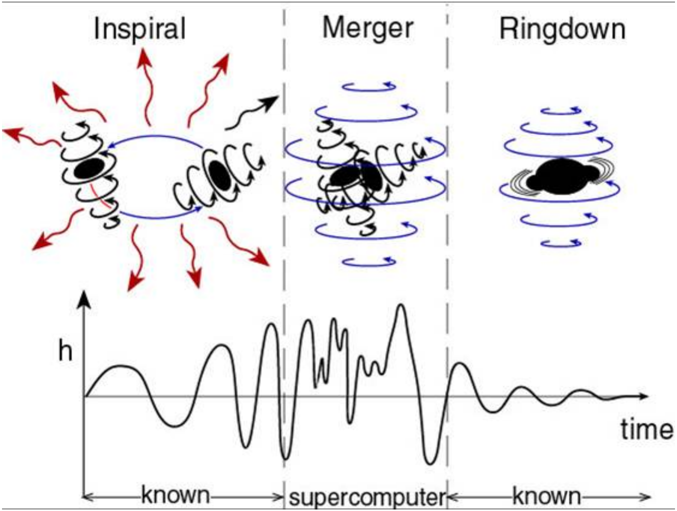
- Use $\rho^{-4}\Psi_4$, $\qquad \rho = -\frac{1}{r - ia\cos\theta}$
- Similarly: $\rho^{-2}\Phi_2$
  (But Ψ_0 and Φ_0)
- $\Psi_4 = \int d\omega\, e^{-i\omega t} \sum_{lm} e^{im\phi} S_{lm}(\theta, a\omega) R_{lm}(r, \omega)$

Black Hole Ringdown



(Figure: Kip Thorne)

Black Hole Ringdown



(Figure: Kip Thorne)

(NR: ADM)

Quasi-normal Modes

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Late times: $$h \sim \sum C_{lmn} e^{-i\omega_{lmn}(t-r_*)} S_{lm}(\theta, a\omega_{lmn})$$

Modes: $$h_{lm} = \sum_{n=0}^{\infty} C_{lmn} e^{-i\omega_{lmn} t} \qquad (Y_{lm} \text{ vs. } S_{lm})$$

Overtones

Modes: $$h_{lm} = \sum_{n=0}^{\infty} C_{lmn} e^{-i\omega_{lmn} t}$$

$$\omega = \omega_{\mathrm{r}} + i\omega_{\mathrm{i}} = \omega_{\mathrm{r}} - i/\tau$$
$$h \sim \cos(\omega_{\mathrm{r}} t) e^{-(t/\tau)}$$

- $n =$ overtone index
- No-hair: $\omega_{lmn} = \omega_{lmn}(M_f, a_f)$
- n sorts by decreasing damping times
- Increasing $n \quad \rightarrow \quad$ lower frequency
- overtones often ignored (“subdominant”)

Ringdown Waveform Modeling

- Buoannano, Cook, Pretorius (2007): equal mass BBH
  - (2,2,0) + 3 overtones good even before peak of Ψ_4
    - $t(\Psi_{4,\text{peak}}) \sim t(h_{\text{peak}}) + 10M$

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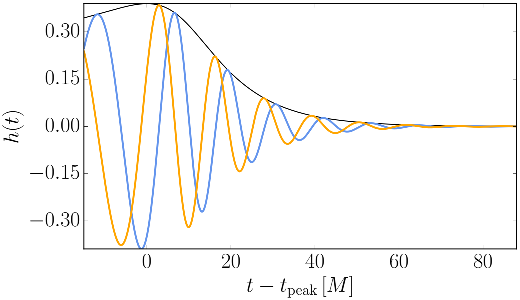
- EOB ringdown modeled with QNMs including overtones
- Community: QNMs good for modeling, but h still non-linear at t_{peak}

Ringdown Start Time

At what point do QNMs provide the correct description?



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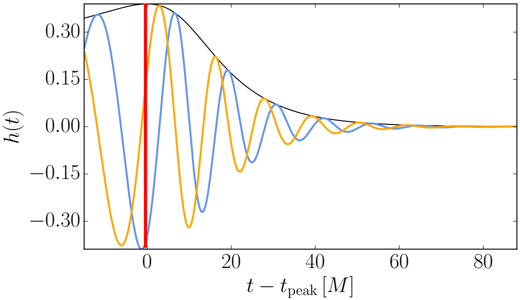
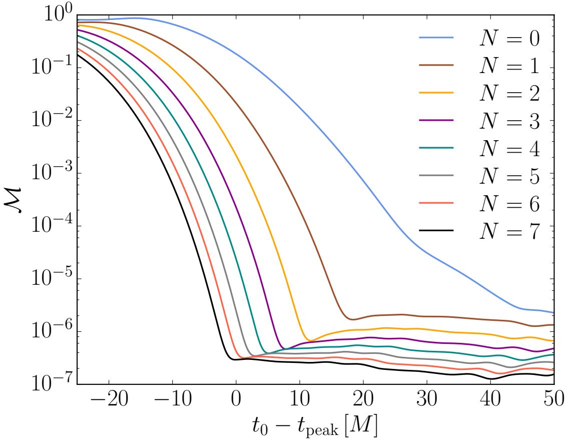


At t_{peak} (or even before) by including overtones!

(Giesler, Isi, Scheel, Teukolsky 2020)

$$h_{22} = \sum_{n=0}^{N} C_{22n} e^{-i\omega_{22n}(t-t_0)}$$

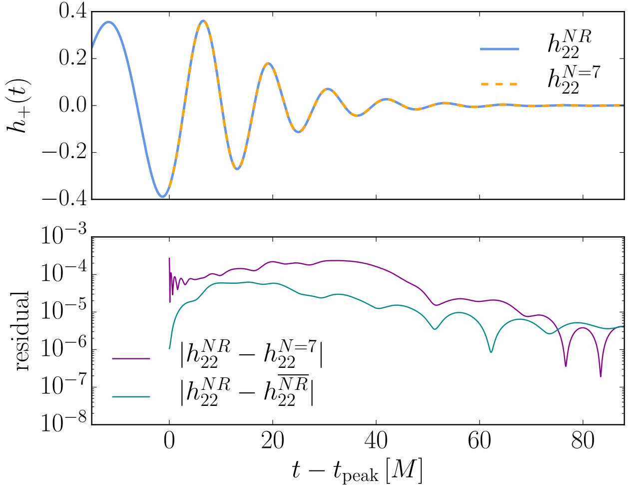
Least-squares $\rightarrow$ C_{22n}, $(M_f^{\rm NR}, a_f^{\rm NR}) \rightarrow \omega_{22n}$



$$\mathcal{M} = 1 - \frac{\langle h_{22}^{\rm NR}, h_{22}^{N}\rangle}{\sqrt{\langle h_{22}^{\rm NR}, h_{22}^{\rm NR}\rangle \langle h_{22}^{N}, h_{22}^{N}\rangle}} \qquad \langle x(t), y(t)\rangle = \int_{t_0}^{T} x(t)\overline{y(t)}\, dt$$

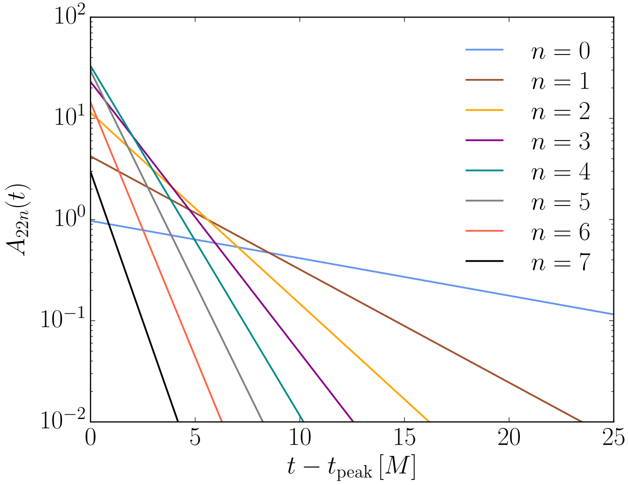
Non-Linearities are Small!

Overtones $\rightarrow$ linear description



h_{22}^{NR} = SXS:BBH:0305

Overtone Decomposition



- Fundamental not dominant until $\sim 10M$ (GW150914: ~ 3 ms)
- Early part dominated by overtones, not non-linearities!

- Other evidence: small perturbations of ω's make fit worse
- Controversy! (e.g., QNMs overcomplete, unstable, . . .)

Fitting Exponentials

$$Ae^{i(\omega t+\phi)} = Ae^{i\phi}e^{i\omega t} = Ze^{i\omega t}$$

- ω known: Linear least squares
  - But which modes?

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- ω known: Linear least squares
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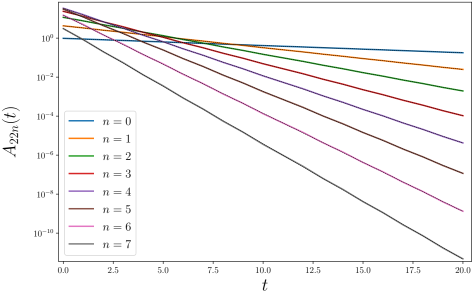
- ω not known: Ill-conditioned   e.g. Lanczos (1956)
  - "Best" algorithm: Variable Projection (VARPRO)

    Golub & Pereyra (1973)

Ill-Conditioning (Linear Case)



Analytic

LLSQ: $[t_0, 80M]$
$t_0 \in [0, 20M]$, $\Delta t_0 = 1M$

Machine precision $\sim 10^{-16}$

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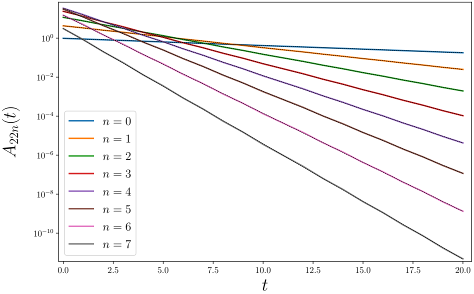

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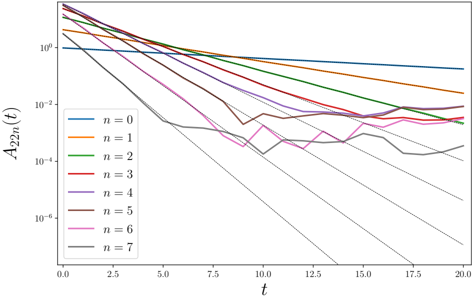
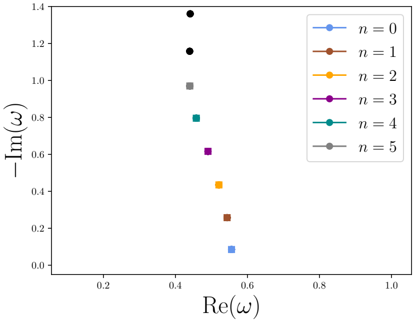


Noise 10^{-5}

A's not recovered at $\sim 10^{-2} - 10^{-3}$

Ill-Conditioning (Nonlinear Case)



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$t_0 \in [0, 10M]$, $\Delta t_0 = 1M$

One mode at a time

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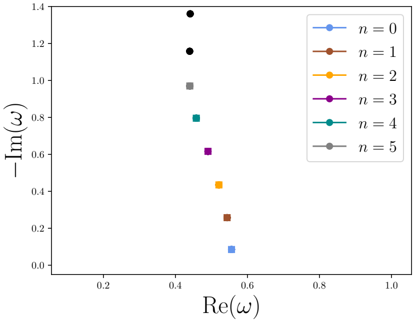

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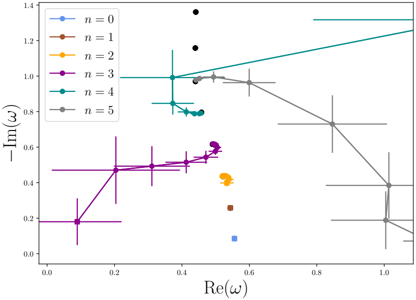

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Noise Sources

Theoretical investigations with NR vs Detection via data analysis

- Nonlinearities
  - Quadratic modes arXiv:2208.07374, arXiv:2208.07380
- Unmodeled modes
- Numerical error
- Frame dependence (BMS . . .)
- Precession

Summary

- Ringdown begins close to peak strain
- Overtones dominate early ringdown
- Fitting can be tricky! (VARPRO, including all modes . . .)
- Exciting new results on the way!

- Non-linearities in the ringdown surprisingly small

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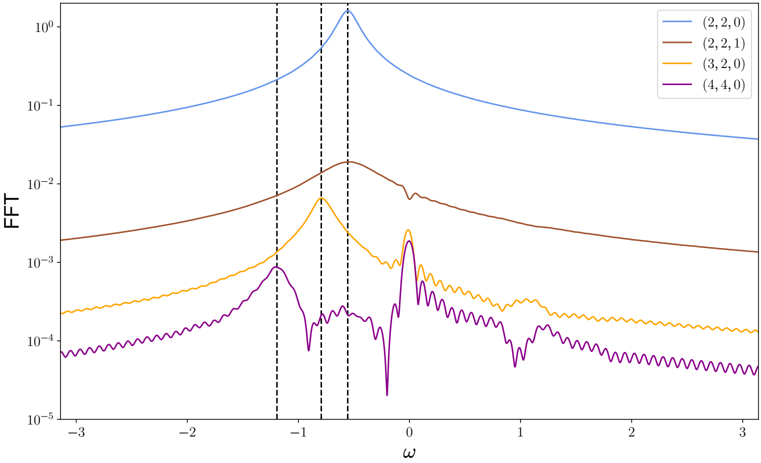
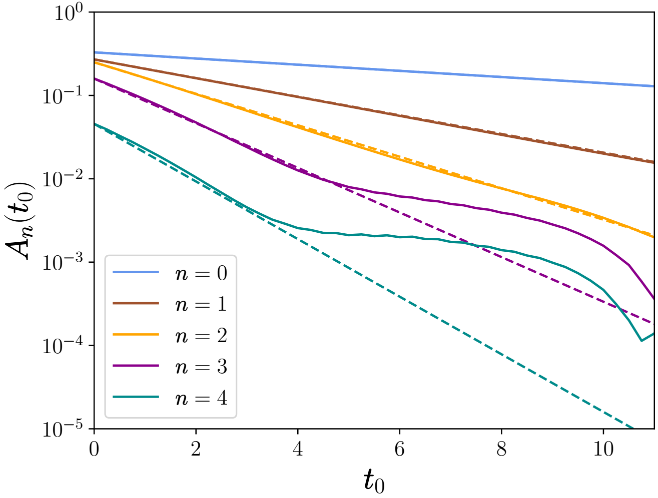
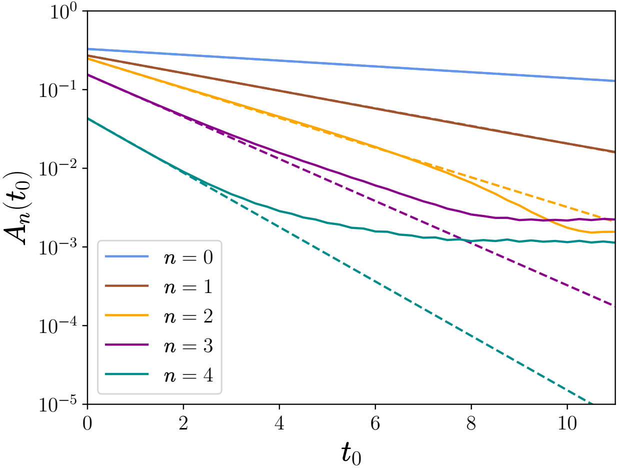
Filtering the 22 Mode



- Spherical-spheroidal mixing: (3,2,0) & (3,2,1)
- BMS mixing: (4,4,0)



- Filter spherical-spheroidal and BMS mixing modes
- Filter all $(2, 2, n)$, $n > 4$
- LLSQ for amps as fn of t_0



- Analytic model $(2, 2, n)$, $n \leq 4$
- Gaussian noise, amp $\sim 10^{-5}$
- Conclusion: nonlinearities, unmodeled modes, numerical error likely explain deviations in NR fitting

Excitation Factors

$$\psi(\omega, \mathbf{x}) \sim \int G(\mathbf{x}, \mathbf{x}', \omega) T(\mathbf{x}', \omega)\, d^3x'$$

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- Giesler et al: ratios of E_{22n}, $n \leq 5$ agree well with C_{22n}

methods of Zhang et al (2013)

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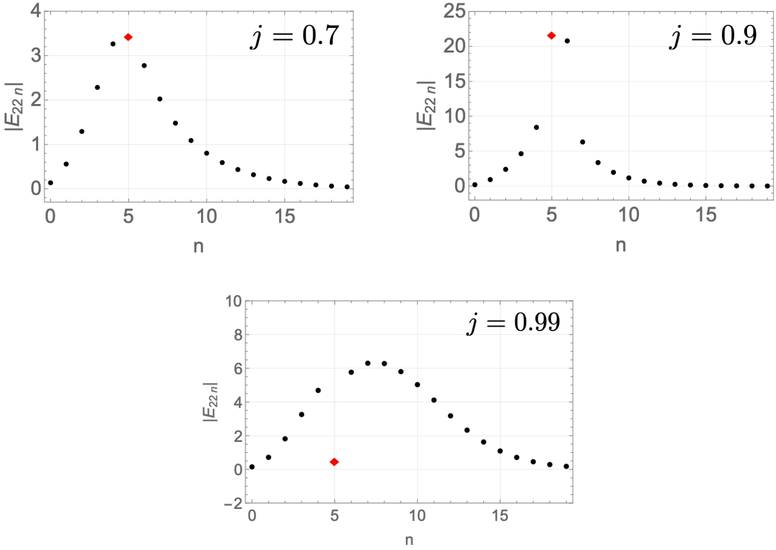
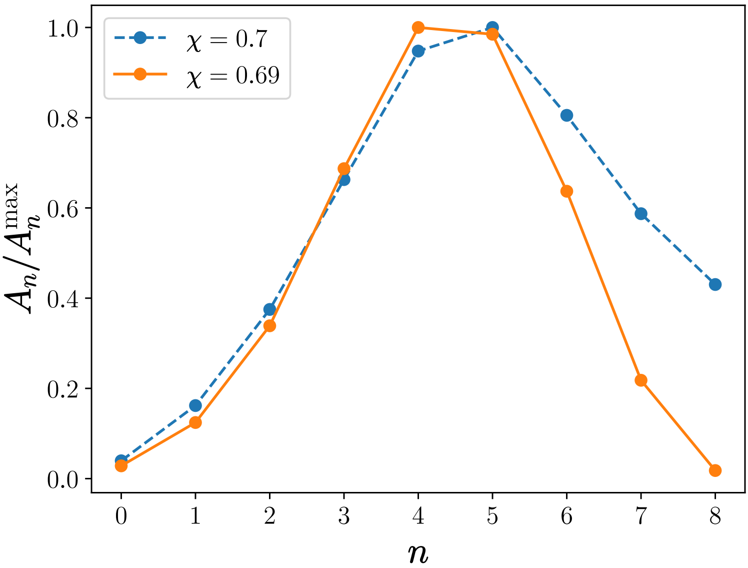
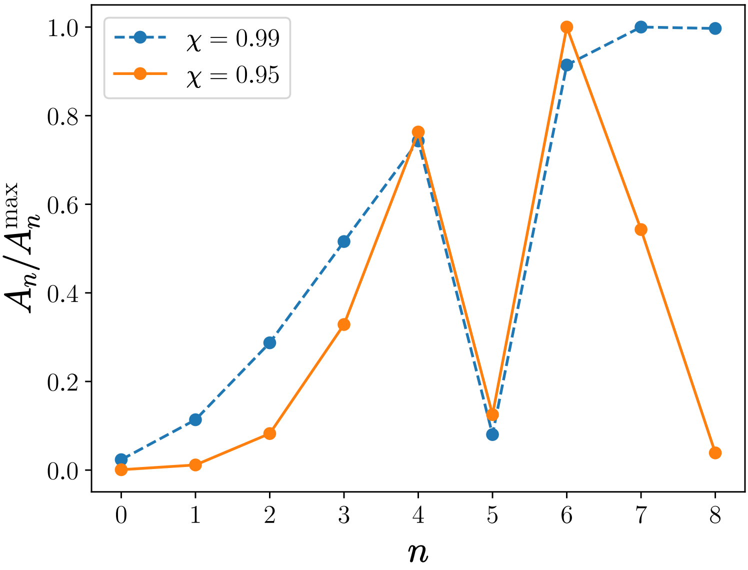


Oshita (2021)



- LLSQ using NR (M, χ) (no filtering)



$\chi = 0.99$
$\chi = 0.95$
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
$A_n/A_n^{\max}$
n

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