UNIVERSITY OF MARYLAND
Department of Physics
College Park, Maryland

PHYSICS Ph.D. QUALIFYING EXAMINATION
PART I

August 27, 2015 9:00 a.m. – 1:00 p.m.

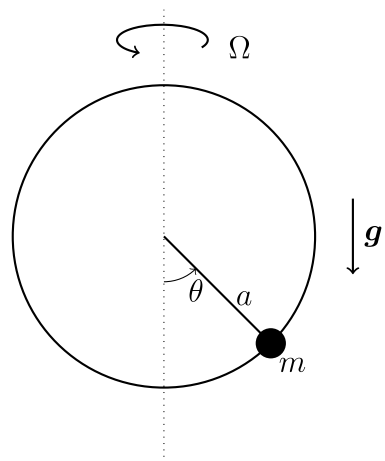
Do any four problems. Each problem is worth 25 points. Start each problem on a new sheet of paper (because different faculty members will be grading each problem in parallel).

Be sure to write your Qualifier ID ("control number") at the top of each sheet — not your name! — and turn in solutions to four problems only. (If five solutions are turned in, we will only grade # 1 - # 4.) For whichever problem (or problems) you skip, fill in a placeholder form so that the grader for that problem will have something from every student.

You may keep this packet with the questions after the exam.

Problem I.1

A point mass m is constrained to move on a massless hoop of radius a that rotates about its vertical symmetry axis with constant angular speed Ω. The only external force is gravity, with acceleration $\boldsymbol{g}$. (see Figure)



(a) [**6 points**] Find the Lagrangian. Show that the Euler-Lagrange equation of motion is of the form:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{a}\sin\theta\left(1 - A^2\cos\theta\right)$$

where you must define the constant A in terms of g, m, a, and Ω. The rest of this problem can be done even if you do not know A.

(b) [**3 points**] Note that $\theta = 0$ is an equilibrium. Under what conditions is there another equilibrium with nonzero $\theta = \theta_0 < \pi$?

(c) [**5 points**] Assuming there is an equilibrium at $\theta_0 > 0$, expand the Euler-Lagrange equation about θ_0 and find the frequency of small oscillations, expressed in terms of A, g, m, a and/or Ω (not involving θ_0). Is this equilibrium stable or unstable?

(d) [**5 points**] Consider the special case $A = 1$. Check that $\theta = 0$ is the only stable equilibrium. Expand the Euler-Lagrange equation about $\theta = 0$ and find a nonlinear ODE for $\theta(t)$ for small oscillations (keep small terms only up to lowest non-vanishing order).

(e) [**6 points**] Suppose $\dot{\theta}(0) = 0$ and $\theta(0) = \varepsilon \ll 1$. From your ODE, find the period of oscillation. Leave your answer in terms of a *unitless* definite integral multiplying some combination of physical parameters.

Problem I.2

Consider an infinite sheet of charge per unit area σ in the $x-y$ plane, located at $z=0$. The sheet oscillates in-plane along the x direction with a displacement $\Delta x = Re[d\, e^{-i\omega t}]$. Maxwell's equations in CGS units are given below; use these units throughout.

(a) [**5 points**] If the charged sheet is stationary ($d=0$), calculate/describe the magnitude and direction of the electric field from the sheet.

(b) [**5 points**] Now consider the oscillating sheet ($d \neq 0$) at a point in time when the sheet is moving. Make a sketch of the $z-x$ plane, sketching electric field lines, and indicating the direction of the magnetic field at three or more relevant points. What is the time-averaged direction of the Poynting vector? How do the magnitudes of **E** and **B** fall off with distance $|z|$ from the charged sheet?

(c) [**5 points**] Starting from Maxwell's equations, show that the time varying electric field E satisfies a differential equation of the form

$$\frac{\partial^2}{\partial z^2}E + a^2 E = b\delta(z),$$

where $\delta(z)$ is the Dirac delta function and the constants a and b are functions of d, c, ω, and σ. Give explicit expressions for a and b. [If you don't solve this part, you can still go on and solve parts (d) and (e) in terms of the constants a and b.]

(d) [**5 points**] What is the solution to the equation above for $z>0$? For $z<0$? Solve the differential equation for the time varying electric fields. Use the matching condition at $z=0$ to determine the amplitude of the oscillating electric fields on either side of the sheet. Also find the magnetic field amplitude.

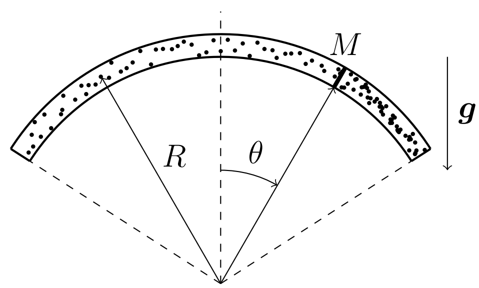
(e) [**5 points**] Calculate the rate at which energy per unit area is radiated from the sheet. Where does this energy come from? Show that the energy radiated balances the energy input.

Maxwell's equations in Gaussian units:

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

Problem I.3

An airtight tube of cross-sectional area A, bent in a circular arc of radius R and having a total angular spread of 2 radians, is shown in the figure below. The radial width of the tube is much less than R. In the tube, a piston of mass M and negligible thickness is subject to gravitational acceleration $\boldsymbol{g}$. The angular position of the piston is θ (in radians). The dots represent ideal gas molecules of which there are N on each side of the piston (neglect the effects of gravity on the gas). The entire system is enclosed in a heat bath at temperature T. For high T, the high pressure on both sides of the piston forces it to $\theta = 0$. The theme of this problem is to explore how the equilibrium position of the piston changes as T decreases.



(a) **[3 points]** What are the volumes of the left and right chambers, V_L and V_R, of the bent tube? And what is the gravitational potential energy of the piston? (Since the radial width is assumed small, you can neglect any curvature corrections.)

(b) **[5 points]** Recall that the partition function Z_1 of an ensemble of free particles in 3 dimensions is V/ℓ_Q^3, where the quantum length or thermal wavelength $\ell_Q = h/(2\pi m k_B T)^{1/2}$. Obtain the partition function $Z(T, \theta)$ for the system (piston plus both chambers of gas) when θ is considered fixed. (You may assume that the gas particles are indistinguishable in each chamber; the eventual solution is not changed if they are distinguishable.)

(c) **[3 points]** Obtain the free energy $F(T, \theta)$ for the system.

(d) **[6 points]** Verify that $\theta = 0$ is always an extremum of $F(T, \theta)$ at fixed T. Show that two new extrema emerge below a transition temperature T_c. Find T_c in terms of the parameters given in the description of the system.

(e) **[5 points]** For T slightly below T_c, show that $|\theta| \propto (T_c - T)^\beta$ for the new extrema, and find the critical exponent β.

(f) **[3 points]** You could also show (but do not need to do so here) that $\partial^2 F/\partial\theta^2|_{\theta=0} = 2Nk_B\,(T - T_c)$. What is the physical implication of this result?

Problem I.4

The "supersymmetric" extension of the Standard Model (SM) entails adding a new particle for every SM particle (called a "superpartner" or "sparticle"). These sparticles, denoted by adding a tilde $\tilde{}$, have the same charges as the SM ones, but greater masses. In this problem, you will get a feeling for how the masses of these sparticles can be determined using measurements done on the SM particles into which they decay.

Consider the decay of a squark $\tilde{q}$ (superpartner of a quark) into the corresponding quark q and a Zino $\tilde{Z}$ (superpartner of the Z boson):

$$\tilde{q} \quad \rightarrow \quad q + \tilde{Z}$$

Denote the sparticle masses by $M_{\tilde{q}}$ and $M_{\tilde{Z}}$, respectively, and treat the quark q as massless, compared to the more massive sparticles.

(a) [**8 points**] In the reference frame where $\tilde{q}$ is *at rest*, find the energy of the quark q, in terms of the masses given above and c, the speed of light in vacuum. Call this energy E_q^{rest}.

(b) [**4 points**] Find the energy (in terms of the given masses) of the $\tilde{Z}$ in the *rest frame* of $\tilde{q}$.

(c) [**3 points**] Suppose that the $\tilde{Z}$ produced from above process decays with a mean proper lifetime τ (this is the value in the rest frame of $\tilde{Z}$). Find the mean decay *length* of $\tilde{Z}$ decay in the rest frame of $\tilde{q}$, in terms of the given masses and τ.

(d) [**4 points**] Suppose the squark $\tilde{q}$ is moving in the *laboratory frame* with speed $\beta_{\tilde{q}}$ (in units of c) and let θ be the angle between its velocity and that of q in the rest-frame of $\tilde{q}$. Find the energy E_q^{lab} of the quark q in the laboratory frame in terms of its energy E_q^{rest}, $\beta_{\tilde{q}}$ and θ.

(e) [**3 points**] Find the location of the *endpoints* (i.e., maximum and minimum values) of the energy spectrum of the quark q in the laboratory frame in terms of E_q^{rest} and $\beta_{\tilde{q}}$.

(f) [**3 points**] Find the expression for the *geometric* mean of the above two endpoints and, using the result of part (a), express this in terms of the given masses $M_{\tilde{q}}$ and $M_{\tilde{Z}}$ only.

Thus, measuring the endpoints of the quark's energy spectrum in the laboratory frame and assuming simply that the squarks all had the same speed in the laboratory frame (but *not* knowing that speed), we can determine a combination of the two unknown masses, $M_{\tilde{q}}$ and $M_{\tilde{Z}}$.

Problem I.5

A uniform string of length L under tension τ undergoes small transverse oscillations. The mass per unit length of the string is given by μ, and the equilibrium position of the string lies along the x axis. The transverse displacement of the string at the point with coordinate x at time t is denoted by $y(x,t)$. One end of the string at $x = 0$ is attached to a fixed support so that the transverse displacement at this point vanishes, $y(0,t) = 0$. The other end of the string is attached to a point particle of mass m that is restricted to lie along the line $x = L$, but is free to move without friction along the y direction.

(a) [**2 points**] Write down the wave equation of motion for small amplitude displacements $y(x,t)$. Express the velocity of propagation of transverse waves in terms of τ and μ.

(b) [**5 points**] By applying Newton's 2$^{\text{nd}}$ Law to the mass m, show that the appropriate boundary condition for *small* displacements along y at $x = L$ has the form

$$\kappa \frac{\partial y}{\partial x} = -\frac{\partial^2 y}{\partial t^2} \, . \tag{1}$$

Express the constant κ in terms of the physical parameters in the problem.

(c) [**10 points**] Use the boundary condition above to obtain a transcendental equation that implicitly determines the characteristic frequencies of the normal modes of this system. (You may write the equation in terms of a wavenumber k instead of a frequency parameter).

Note: If you can't get the answer to this part, you can still answer parts (d) and (e) through other lines of reasoning for partial credit.

(d) [**4 points**] Use this transcendental equation to obtain the solution for the wavelengths of the normal modes in the limit that $m \to \infty$, (or, more precisely, $m \gg \mu L$). Give a physical interpretation of your result.

(e) [**4 points**] Use the equation from part (c) to obtain the solution for the wavelengths of the normal modes in the limit that $m \to 0$. Give a physical interpretation of your result.