UNIVERSITY OF MARYLAND
Department of Physics
College Park, Maryland

PHYSICS Ph.D. QUALIFYING EXAMINATION
PART I

August 28th, 2014   9:00 a.m. – 1:00 p.m.

Do any four problems. Each problem is worth 25 points.

Put all answers on your answer sheets.

Be sure your Qualifier ID Number is at the top right corner of each sheet and turn in solutions to four problems only. (If five solutions are turned in, we will only grade # 1 - # 4.)

Problem I.1

A particle of charge q and mass m moves in crossed magnetic and electric fields. The fields are given in component form $\boldsymbol{E} = (xE_0', 0, 0)$ and $\boldsymbol{B} = (0, 0, B_0)$ where E_0' and B_0 are constants.

(a) **[2 points]** First consider two special cases: Briefly discuss the general particle motion if only $\boldsymbol{E} \neq 0$, and if only $\boldsymbol{B} \neq 0$.

(b) **[5 points]** Returning to the general case, write the Lagrangian for this system. This will require constructing potentials Φ and $\boldsymbol{A}$. Pick a gauge where the potentials depend only on the variable x.

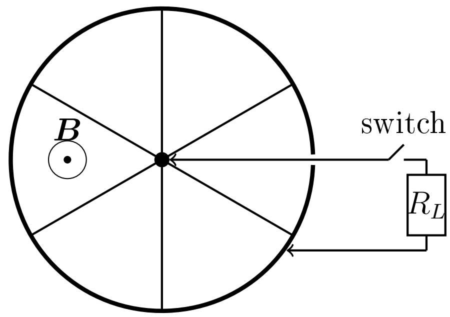
(c) **[3 points]** Find the conditions for an equilibrium in which the particle is at rest.

(d) **[6 points]** Obtain the equations for oscillations about this equilibrium. Be careful, because of the linear dependence of the Lagrangian on velocity this system does not fit the usual pattern.

(e) **[5 points]** Find the normal mode frequencies ω of these oscillations.
What stability condition for the equilibrium of part (c) follows from the expression for ω?

(f) **[4 points]** How many different normal modes are there (in total)? How many go with $\omega^2 = 0$?
Explain the physical origin of the $\omega^2 = 0$ solutions.

Problem I.2



A flywheel with thin spokes, as shown in the figure, has outer radius a, moment of inertia I, and is made entirely of electrically conducting material. There is a uniform magnetic field $\boldsymbol{B}$ normal to the plane of the wheel. The wheel is set into rotation on frictionless bearings at angular velocity ω_0 and then allowed to coast.

Leads are placed in frictionless contact with the center and the rim of the wheel, and a circuit with a switch is set up as shown. The total resistance of the wheel center and rim contacts and the connecting members when static is R_W, and there is a load resistance R_L in the external part of the circuit.

Address each of the questions below with your answers given in terms of ω_0, a, I, B, R_W and R_L.

(a) **[8 points]** Consider mechanical equilibrium of a charged particle on a rotating spoke to determine the electric potential difference across the open switch.

(b) **[5 points]** With the switch closed, what is the total power dissipated in all parts of the circuit?

(c) **[8 points]** With the switch closed, what is the angular acceleration rate of the wheel at instantaneous rotation rate $\omega(t)$?

(d) **[4 points]** Find $\omega(t)$ for $t > 0$ if the switch is closed at $t = 0$.

Problem I.3

Radiation in a cavity can be treated as a gas of ultrarelativistic particles. The particles have momentum $\mathbf{p}$ and energy ε, which are related by $\varepsilon = pc$.

(a) **[10 points]** Show that the pressure of this gas is $P = u/3$, where u is the energy density.

(b) **[5 points]** Derive the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$.
[Hint: Use the Helmholtz free energy $U - TS$.]

(c) **[5 points]** Assuming that the energy density is completely determined by the temperature alone, that is, that $u = u(T)$, use the results of parts a) and b) to derive the Stefan-Boltzmann Law: $u(T) = \sigma T^4$, where σ is a constant.

(d) **[5 points]** Show that when radiation, contained in a vessel with perfectly reflecting walls, is compressed adiabatically, it obeys the equation $PV^\gamma =$ constant, where V is the volume of the gas.
Determine the value of γ.
[Hint: use the First Law].

Problem I.4

Some years ago it was suggested that an interstellar drive could be achieved by directing a beam from an earth-bound laser at a spacecraft, which itself would carry no fuel and which would be propelled due to radiation's momentum transfer. Large speeds apparently would be reached.[1] This problem explores the relativistic kinematics of two versions of this approach.

(a) **[2 points]** In the first version, the laser beam is totally absorbed by the spacecraft, which starts from rest with rest mass m_0 and accelerates by absorption of laser light to a speed $\beta = \frac{v}{c}$. Show that the total momentum four vector transferred to the spacecraft in this case is a null vector.

(b) **[8 points]** Show that the spacecraft's rest mass after absorption of the laser beam is given by

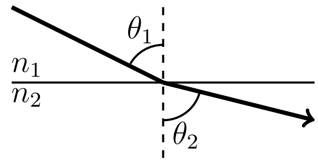
$$m = \sqrt{\frac{1+\beta}{1-\beta}}\, m_0$$

(c) **[8 points]** Now consider version two, where the laser beam is fully reflected by a mirror on the spacecraft in such a way that the rest mass of the spaceship does not change. Working in the rest frame of the laser, find the relation between final velocity β of the spaceship and fractional energy loss by the laser beam.

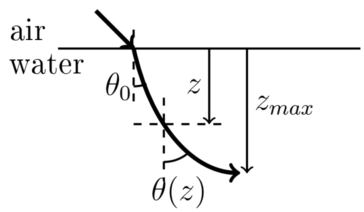
(d) **[7 points]** For version two, evaluate the efficiency per photon ϵ of this system: $\epsilon = \frac{dE}{E_\gamma^{in}}$ where dE is the energy gained by the spaceship moving with speed β, and E_γ^{in} is the energy of one incident photon.
Show that $\epsilon \to 1$ as $\beta \to 1$

[1] However, stopping would be a problem...

Problem I.5



(a) **[6 points]** Derive the relationship between the angle of incidence and the angle of refraction for a wave passing through a boundary between two media with different group velocities, c/n_1 and c/n_2, using Fermat's principle (principle of least time).



(b) **[4 points]** Neglecting the effects of temperature variations, we may assume that (due to increasing pressure) the speed of sound in water, $v(z)$, is a monotonically increasing function of the depth z. In this case, $n(z) = v(0)/v(z)$ is a monotonically decreasing "index of refraction" for sound. Consider a sound ray obliquely projected into an infinitely deep body of water as shown in the figure above. Find the angle $\theta(z)$ that the sound ray makes with the vertical at a depth z in terms of the angle θ_0 just beneath the surface. (Neglect diffraction and attenuation throughout this entire problem.)

(c) **[3 points]** Find a relationship between the initial angle θ_0 and the refractive index at the maximum depth z_{max}.

(d) **[4 points]** For the case $n(z) = (1 - \gamma z)^{1/2}$, where γ is a positive constant, find z_{max} explicitly as a function of θ_0.

(e) **[8 points]** By relating the refraction angle $\theta(z)$ and tangent to the path $\frac{dz}{dx}$, show that the trajectory of the "sound ray" is a parabola, for the form of $n(z)$ given in part (d).